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Reliability of an undrained clay slope formed from spatially random soil

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ABSTRACT: The majority of slope stability analyses are deterministic in nature in that the inherent variability of the soil is not modelled directly, rather some 'average' soil strength value is assumed which leads to a Factor of Safety. Slope stability analyses obtained through slip circle or more recently finite element methods allow the inclusion of strata and defined soil regions but have not taken account of the spatial variability and correlations within the soil mass. In the present study, the spatial correlation effect has been studied in detail in the analysis of a cohesive slope formed from undrained clay. The model has involved a combination of random field theory with an elasto-plastic finite element algorithm. Monte Carlo simulations have been performed in order to assess the influence of the variance of the soil shear strength and its spatial correlation length on the stability of the slope. The results of this parametric study enable the traditional Factor of Safety of the slope to be re-interpreted in the context of Reliability Based Design.

1 INTRODUCTION

The majority of slope stability analyses and designs are deterministic in nature based on a Factor of Safety approach that takes into account the material parameters and slope geometry. This Factor of Safety is usually based on both design codes and a degree of engineering judgement which leads to the possibility that different Factors of Safety could be suggested by independent designers. Standard procedures published by Bishop (1955), Bishop and Morgenstern (1960), and Morgenstern and Price (1965) using slip circles are still commonly used in industry with computers used to obtain the minimum Factor of Safety. Although these methods are capable of analysing zoned and stratified soil they are unable to account for the spatially variable nature of soils.

In all analyses there are five distinct sources of uncertainty (Cambou 1975, Mostyn and Li 1993):

i) Material properties, e.g. soil shear strength,

ii) Loading conditions,

iii) Boundary conditions,

iv) Calculation methods (or systematic error),

v) Continuum characteristics.

In the majority of analyses, all of these sources are assumed to be deterministic. Previous studies by Dai *et al* (1993) presented a method of probabilistic slope stability analysis using deterministic slip circle computer software and Monte Carlo simulation. In their study the spatial correlation of the soil parameters was not implemented.

In the present analyses, visco-plastic finite element code has been combined with the Local Average Subdivision (LAS) technique (Fenton 1990, Fenton and Vanmarcke 1990) to perform Monte Carlo simulations of slope stability problems where the soil displays spatial variability. This approach was noted by Mostyn and Li (1993) as being a promising method for performing probabilistic slope stability analyses. Previous studies of probabilistic seepage, settlement, and bearing capacity problems have been presented by the authors using the same methodology (Griffiths *et al* 1994, Paice *et al* 1996, Paice and Griffiths 1997) and the current studies form part of a wider study of probabilistic geotechnical problems conducted by the authors.

2 BRIEF DESCRIPTION OF FINITE ELEMENT AND RANDOM FIELD MODELS

The finite element program used the Visco-plastic algorithm (Zienkiewicz and Cormeau 1974) to model the non-linear soil behaviour and was similar to that published in the text by Smith and Griffiths (1988). In the analyses, the slope was assumed to be formed from undrained clay characterised by the Mohr-Coulomb failure criterion with $\phi_u = 0°$. This assumption restricted the soil parameters to

i) Young's modulus, E,

ii) Poisson's ratio, ν,

iii) Undrained shear strength, c_u,

iv) Unit weight, γ.

To simplify the analyses, the Young's modulus, Poisson's ratio, and unit weight of the clay are assumed to be characterised by deterministic values. The only spatially random material parameters was therefore the undrained shear strength which has been modelled using the LAS technique. In addition to these assumptions, the loading conditions (gravity) and boundary conditions were also assumed to be deterministic.

Measurements of the undrained shear strength of clays have shown coefficients of variation in the range $0.2 \le \sigma_{c_u}/\mu_{c_u} \le 0.5$ with a recommended value of $\sigma_{c_u}/\mu_{c_u} = 0.3$ (Lee *et al* 1983). Since the undrained shear strength may not take negative values, the lognormal probability distribution has been assumed and the undrained shear strength obtained through the transformation

$$c_u^i = \exp\{\mu_{\ln c_u} + \sigma_{\ln c_u} g_i\}, \qquad (1)$$

in which c_u^i is the undrained shear strength assigned to the i^{th} element, g_i is the local average of a standard Gaussian random field, g, over the domain of the i^{th} element, and $\mu_{\ln c_u}$ and $\sigma_{\ln c_u}$ are the mean and standard deviation of the logarithm of c_u (obtained from the 'target' mean and standard deviation μ_{c_u} and σ_{c_u}).

The LAS technique (Fenton 1990, Fenton and Vanmarcke 1990) generates realisations of the local averages g_i which are derived from the random field g having zero mean, unit variance, and a spatial correlation controlled by the scale of fluctuation, θ_{c_u}. As the scale of fluctuation tends to infinity, g_i becomes equal to g_j for all elements i and j – that is the field of undrained shear strengths tends to become uniform on each realisation. At the other extreme, as the scale of fluctuation tends to zero, g_i and g_j become independent for all $i \neq j$ – the undrained shear strength varies rapidly from point to point.

In the two-dimensional analyses presented in this paper, the scales of fluctuation in the horizontal and vertical directions are taken to be equal (isotropic) even though the scale of fluctuation is likely to be greater horizontally than vertically in naturally deposited soil. The two-dimensional model also implies that the out-of-plane scale of fluctuation is infinite – the undrained shear strength is constant in this direction. This is clearly a deficiency of the present model, however it is believed that useful information regarding to the reliability of an undrained clay slope may still be observed from the analyses.

3 SUMMARY OF RESULTS OBTAINED FROM ANALYSES

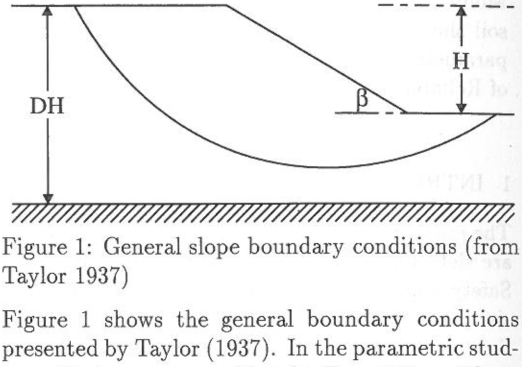

Figure 1: General slope boundary conditions (from Taylor 1937)

Figure 1 shows the general boundary conditions presented by Taylor (1937). In the parametric studies a 45 degree slope of height H = 10.0m with no founding layer was used, i.e. D = 1.0. Monte Carlo simulations using 2000 realisations were performed over the parameter ranges

$$\sigma_{c_u}/\mu_{c_u} \in \{0.2, 0.3, 0.4, 0.5, 0.75, 1.0\}, \qquad (2)$$

where $0.2 \le \sigma_{c_u}/\mu_{c_u} \le 0.5$ is the range coefficients of variation reported by Lee *et al* (1983) and

$$\theta_{c_u}/\mathrm{H} \in \{1/40, 1/10, 2/5, \\ 8/5, 32/5, 128/5, \infty\}, \qquad (3)$$

along with $\mathrm{E} = 1.0 \times 10^5\mathrm{kN/m^2}$, $\nu = 0.3$, and $\gamma = 20.0\mathrm{kN/m^3}$. The undrained shear strength mean, μ_{c_u}, corresponding to a Factor of Safety of F = 1.0 obtained from the finite element code equalled $\mu_{c_u} = 36.95\mathrm{kN/m^2}$ which is close to the value obtained from Taylor's chart of $c_u \approx 33\mathrm{kN/m^2}$. The

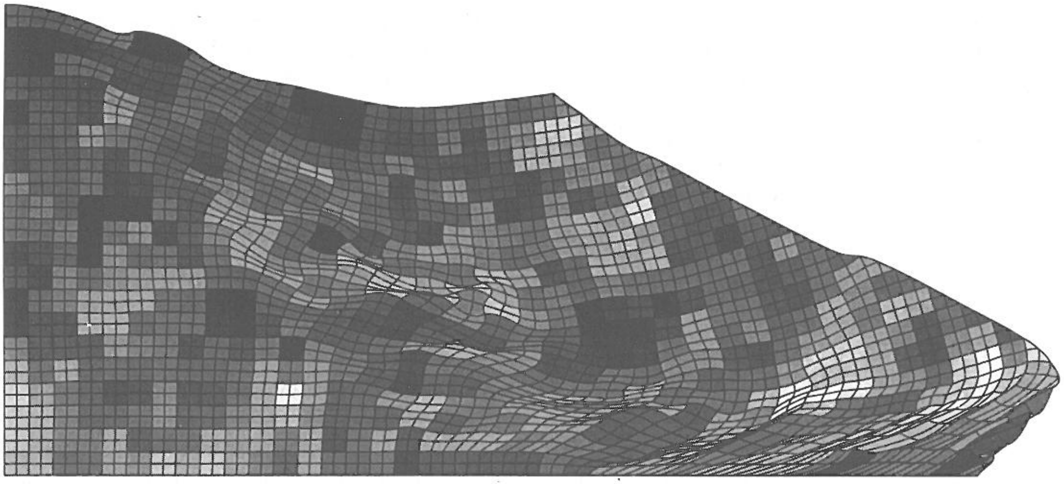

Figure 2: Typical realisation of random undrained shear strength field corresponding to failure for $\sigma_{cu}/\mu_{cu} = 0.5$ and $\theta_{cu}/\mathrm{H} = 1/10$

results of the analyses have been interpreted on a fail/no fail criterion from which the reliability has been assessed. For example, if out of 2000 realisations 200 did not fail and 1800 failed, the reliability would be 10%. Figure 2 shows a typical displaced finite element mesh corresponding to failure of the clay slope for $\sigma_{cu}/\mu_{cu} = 0.5$ and $\theta_{cu}/\mathrm{H} = 1/10$ with the mechanism enhanced by regridding (Griffiths and Kidger 1995). In Figure 2 a global failure mechanism is evident along with a number of local failures in zones of weaker undrained shear strength (lighter greys).

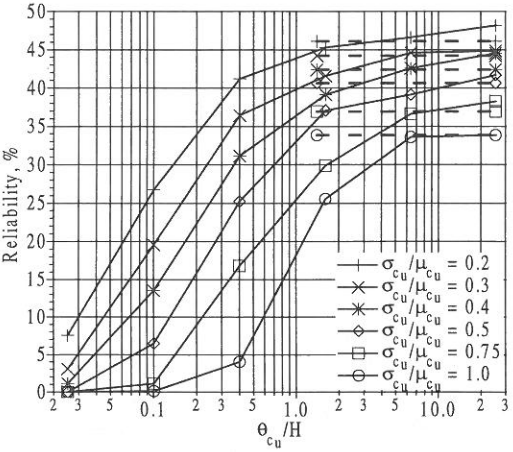

Figure 3: Reliability of earth slope corresponding to $\mu_{cu} = 36.95$ where F = 1.0

Figure 3 shows the variation of the slope reliability corresponding to a undrained shear strength mean of $\mu_{cu} = 36.96\mathrm{kN/m}^2$ (Recall that a uniform slope with this undrained shear strength would have a Factor of Safety of F = 1.0). For all undrained shear strength coefficients of variation, the reliability of the slope increases with greater correlation of the random fields (increasing scale of fluctuation, θ_{cu}). As the scale of fluctuation tends towards infinity, the reliability approaches clear upper bounds.

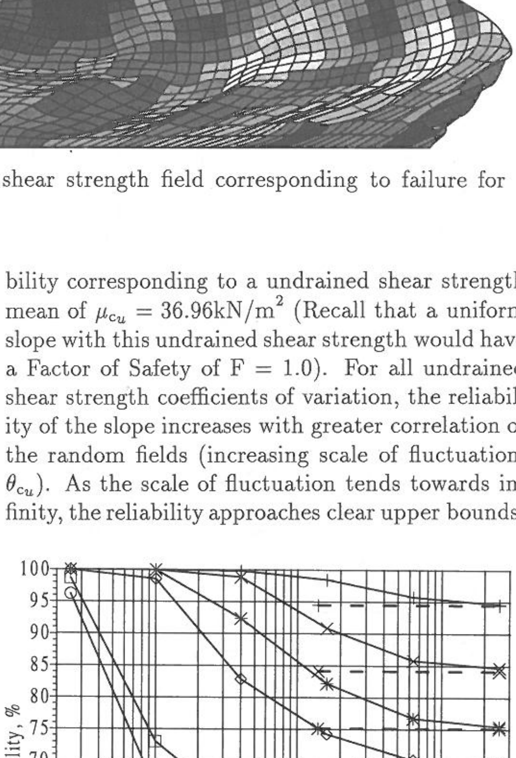

Figure 4: Reliability of earth slope corresponding to $\mu_{cu} = 51.73$ where F = 1.4

Figures 4 and 5 show similar plots for undrained shear strength means of $\mu_{cu} = 51.73\mathrm{kN/m}^2$ and $\mu_{cu} = 97.07\mathrm{kN/m}^2$ corresponding to deterministic Factors of Safety of F = 1.4 and F = 2.6 respectively. Common design Factors of Safety are often in the region of F = 1.5. Clearly at this order of Factor of Safety, Figure 4 shows that the spatial variability of the clay has a significant effect on the reliability of the slope – reliability decreases for increasing undrained shear strength coefficient of variation and scale of fluctuation.

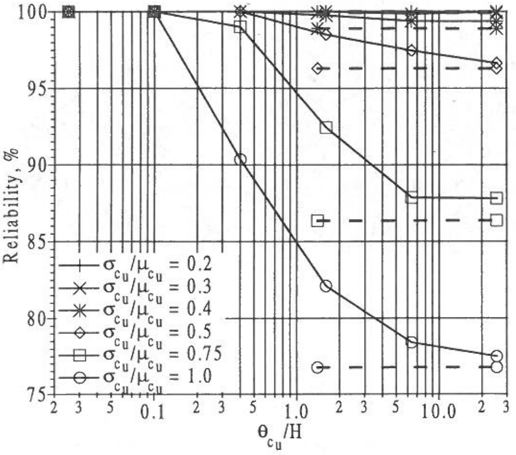

Figure 5: Reliability of earth slope corresponding to $\mu_{cu} = 96.07$ where F = 2.6

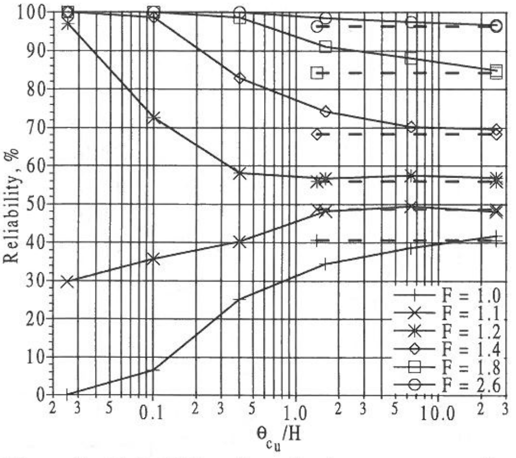

Figure 6: Reliability of earth slope corresponding to $\sigma_{cu}/\mu_{cu} = 0.5$

Figure 6 shows the variation of the slope reliability with undrained shear strength means tuned to the deterministic Factor of Safety for a coefficient of variation of $\sigma_{cu}/\mu_{cu} = 0.5$, the upper bound stated by Lee *et al* (1983). For F $\leq$ 1.1 the reliability of the clay slope increases for rising scale of fluctuation, tending to the upper bounds shown by the dashed plots – the increase of spatial correlation of the undrained shear strength is beneficial to the reliability of the slope. For F $\geq$ 1.2 the opposite behaviour is observed with the increase in spatial correlation reducing the the reliability of the slope. These observation are governed by the relative positions of the lognormal distribution used to model the undrained shear strength and are discussed in the next section.

4 DISTRIBUTIONS OF UNDRAINED SHEAR STRENGTH MEASURES

Figures 7 to 9 show the distributions of the arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 36.95$ where F = 1.0 and $\theta_{cu}/\mathrm{H} \in \{1/40, 2/5, 32/5\}$. For F = 1.0, an increasing proportion of the distribution is located above the undrained shear strength of $c_u = 36.95\mathrm{kN/m^2}$ corresponding to a deterministic Factor of Safety equal to 1.0 (shown by the vertical dashed line) and therefore the reliability of the slope increases. This behaviour corresponds with the plot for F = 1.0 in Figure 6.

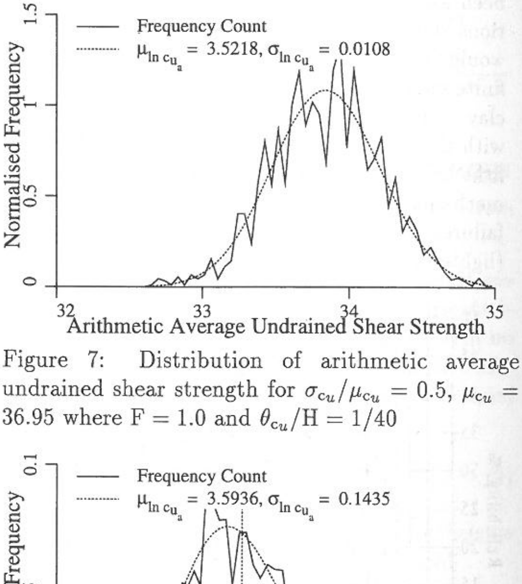

Figure 7: Distribution of arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 36.95$ where F = 1.0 and $\theta_{cu}/\mathrm{H} = 1/40$

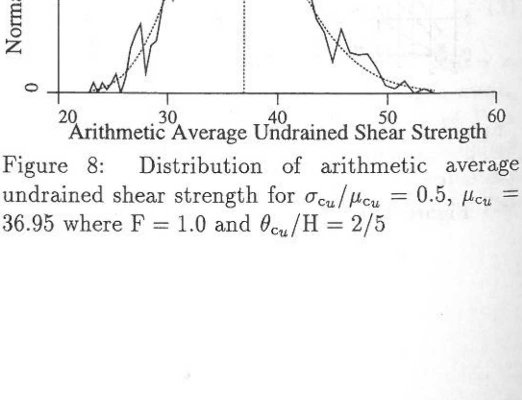

Figure 8: Distribution of arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 36.95$ where F = 1.0 and $\theta_{cu}/\mathrm{H} = 2/5$

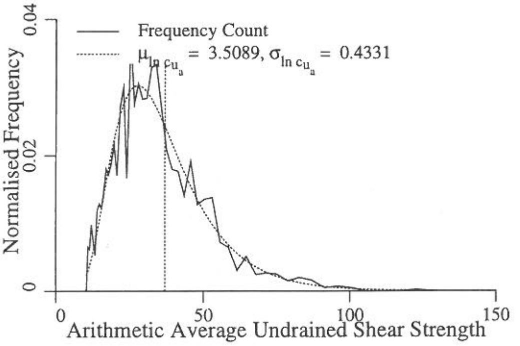

Figure 9: Distribution of arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 36.95$ where F = 1.0 and $\theta_{cu}/\mathrm{H} = 32/5$

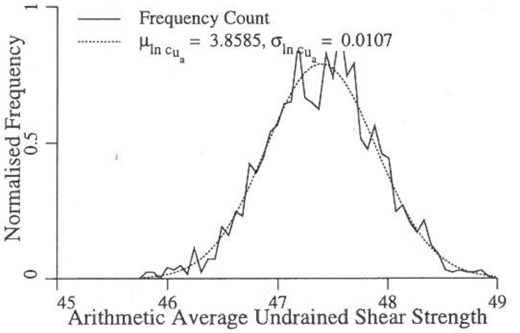

Figure 10: Distribution of arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 51.73$ where F = 1.4 and $\theta_{cu}/\mathrm{H} = 1/40$

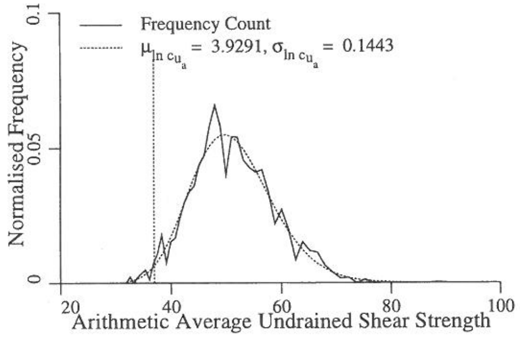

Figure 11: Distribution of arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 51.73$ where F = 1.4 and $\theta_{cu}/\mathrm{H} = 2/5$

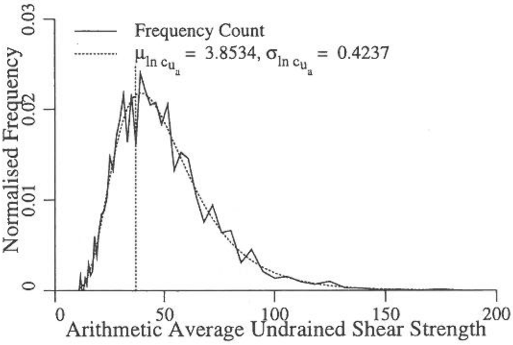

Figure 12: Distribution of arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 51.73$ where F = 1.4 and $\theta_{cu}/\mathrm{H} = 32/5$

Figures 10 to 12 show the distributions of the arithmetic average undrained shear strength for $\sigma_{cu}/\mu_{cu} = 0.5$, $\mu_{cu} = 51.73$ where F = 1.4 and $\theta_{cu}/\mathrm{H} \in \{1/40, 2/5, 32/5\}$. For F = 1.4, a decreasing proportion of the distribution is located above the undrained shear strength of $c_u = 36.95\mathrm{kN/m^2}$ corresponding to a deterministic Factor of Safety equal to 1.0 (shown by the vertical dashed line) and therefore the reliability of the slope decreases. This behaviour corresponds with the plot for F = 1.4 in Figure 6 and explains why there are significantly different behaviours for the various values of F.

Table 1: Reliabilities computed from Monte Carlo simulations and undrained shear strength measures for $\sigma_{cu}/\mu_{cu} = 0.5$ for $\mu_{cu} = 36.95$ where F = 1.0

	Reliability (%)	
θ_{cu}/H	M-C Simulations	Distributions
1/40	0.00	0.00
2/5	25.20	24.93
32/5	38.55	39.71
∞	40.70	38.46

The area underneath the distributions shown in Figures 7 to 12 may be computed to yield the probability of a given event. Computation of average the probabilities of the arithmetic, geometric, and harmonic average undrained shear strengths being greater than $36.95\mathrm{kN/m^2}$ shows close approximation to the reliability of the slope. Table 1 shows a sample of the reliabilities obtained from both the solution of the finite element problem and the averaging of the undrained shear strength quantities. This observation means that for the slope boundary-value problem studied, the reliability of the slope can be obtained without the need to perform the lengthy finite element solutions.

5 CONCLUDING REMARKS

The paper has presented results which form part of a broad study conducted by the authors into the influence of random soil properties on geotechnical design. In this paper, random field methodology has been combined with the finite element method to study the reliability of a slope formed from lognormally distributed undrained clay. Parametric studies to gauge the influence of the undrained shear strength coefficient of variation and spatial correlation have been performed.

For an undrained shear strength mean tuned to a deterministic Factor of Safety equal to 1.0 the reliability of the slope increases for increasing spatial correlation. In addition, rising undrained shear strength coefficient of variation reduces the reliability of the slope as would be expected. For undrained shear strength means tuned to deterministic Factors of Safety of 1.4 or greater (typical deterministic Factors of Safety used for design) the increase in spatial correlation of the undrained shear strength random fields was detrimental to the stability of the slope. These trends correspond with the locations of the undrained shear strength distributions and are the subject of further investigations.

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