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# Accurate numerical prediction of bearing capacity of cohesionless soils

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ABSTRACT: In this paper observations on the computation of the bearing capacity factor $N_\gamma$ by finite elements are presented using a simple linear-elastic perfectly plastic Mohr-Coulomb constitutive soil model. The results of the analyses show that, for this type of stress-strain law, the value of $N_\gamma$ is independent of the footing width provided that a sufficient number of Gauss points are used directly under the footing and a correction is made for the surcharge component of the bearing capacity at the Gauss point depth. When simulating a smooth footing the predicted values of $N_\gamma$ seem to be close to the values suggested by Hansen & Christiensen (1969).

## 1 INTRODUCTION

It is well known that the bearing capacity factor $N_\gamma$ for long strip footings decreases with increasing width of foundation $B$. Evidence of this can be traced back to the well known paper by de Beer (1965) summarised in Figure 1. The two mains reasons proposed for this phenomenon are: (i) effect of the higher foundation pressures reducing the friction angle $\phi$ and (ii) progressive failure producing a grain size effect due to shear band formation. For a particular foundation two other factors can also be considered, (iii) pre-loading of foundation soil and (iv) non-uniformity's of density within the soil.

Hettler and Gudehus (1988) performed triaxial compression and centrifuge tests using Karlsruhe sand and proposed an expression for evaluating the reduction in $\phi$ with confining pressure and its effect on the bearing capacity factor $N_\gamma$. They found that $N_\gamma$ was strongly influenced by the pressure dependence of the friction angle $\phi$. Kimura et al (1985) had previously reported that progressive failure and strong anisotropy in dense sands was also responsible for variations in $N_\gamma$.

Often numerical studies of ultimate bearing capacity (Griffiths 1982 and Kay & Legein 1994) using simple constitutive soil models indicate a reduction in $N_\gamma$ with footing width; although for apparently quite different reasons to those mentioned in the first paragraph. The work presented in this paper will show that this reduction is due to the mesh configuration and once accounted for constant values of $N_\gamma$ can be computed regardless of the footing width. These values can then be compared to theoretical expressions based on plasticity theory proposed by other researchers. However, there are problems involved in the theoretical determination of $N_\gamma$ using plasticity theory due to the curved nature of the slip mechanisms, making the evaluation of $N_\gamma$ very difficult. Of the many solutions available Terzaghi's [6] values are often used, or those suggested by Brinch Hansen (1968) & Meyerhof (1963),

$$N_\gamma = 1.80(N_q - 1)\tan\phi \quad \text{(Brinch Hansen 1968)} \quad (1)$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi) \quad \text{(Meyerhof 1963)} \quad (2)$$

Eurocode 7 Part 1 suggests using the values obtained by Vesic (1973) when the interface angle between the foundation and the sand $\delta \geq \phi/2$,

$$N_\gamma = 2(N_q + 1)\tan\phi \quad \text{(Vesic 1973)} \quad (3)$$

where,

$$N_q = \exp(\pi\tan\phi)\tan^2(45 + \phi/2) \quad (4)$$

To accurately study the reduction in $N_\gamma$ with footing width more sophisticated soil models must be used which can accurately simulate the pressure

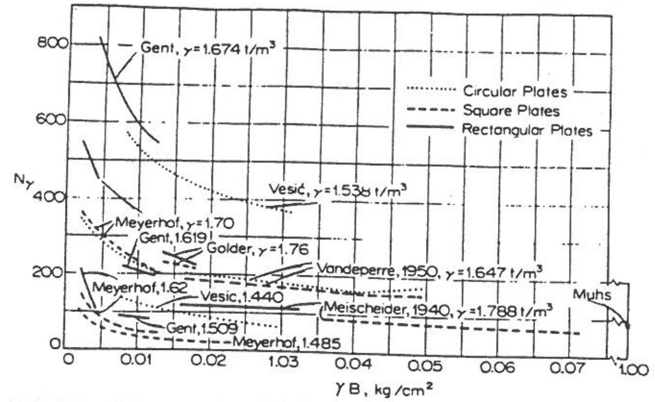


Figure 1 Bearing capacity factor $N_\gamma$ as a function of foundation width $B$ after de Beer (1965)

dependence (i.e. the current isotropic stress level) of the friction angle $\phi$.

## 2 ANALYSIS PROCEDURE

A linear-elastic perfectly plastic Mohr-Coulomb constitutive soil model was used combined with an elastic-viscoplastic stress redistribution algorithm. The soil was attributed the following soil constants $\phi$=25°, $\gamma$=20 kN/m$^3$, $\nu$=0.3, $\psi$=0° (non-associated) and $E$=100 MPa.

Since the problem is symmetrical about the centre line only half of the mesh needs to modelled. To induce bearing failure displacement increments were applied (typically 100) to the nodes at the surface of the soil, representing a smooth footing, and the vertical stresses at the Gauss points directly beneath were averaged to find the bearing stress $q$. At failure $q$ tends to a constant value ($q_{ult}$) and $N_\gamma$ can be determined from Terzaghi's (1943) theory,

$$N_\gamma = \frac{2q_{ult}}{\gamma B} \tag{5}$$

In problems of this type displacement increments must be used as the footing cannot support any normal stress at its edge. In fact, directly adjacent to the edge of the footing the shear strength of the granular soil is zero which tends to induce numerical difficulties due to the development of a singularity. This can often lead to substantial increases in computational effort with increasing friction angle. Also, convergence becomes increasing difficult as the friction angle increases, this is especially the case if rough footing are simulated. In all the analyses performed eight noded quadrilateral elements were used with a 2x2 integration rule. The initial stresses were set by simply multiplying the vertical distance of the Gauss point from the surface by the unit weight for stresses acting in the vertical direction, and then by $K_o$ for stresses acting in the horizontal direction (all runs $K_o$ =1.0).

## 3 PRELIMINARY STUDIES

In these initial studies the mesh shown in Figure 1 was used for all the analyses. The width of the footing was varied by simply increasing the number of nodes used to represent the footing. This means that an increasing number of Gauss points are used to find $N_\gamma$.

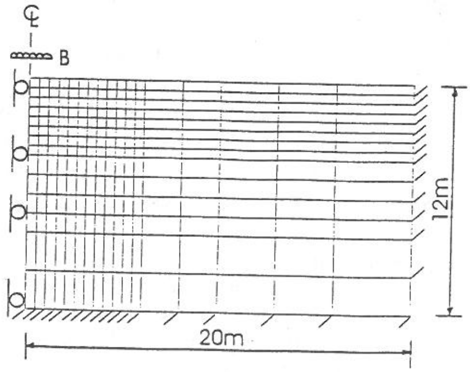


Figure 2 Finite element mesh

The results of the computer runs are shown in Figure 3. It appears that $N_\gamma$ is a function of the footing width, even at a friction angle of $\phi$=25°. Using finite elements reductions in $N_\gamma$ with footing width of this type have been reported by several researchers. If higher friction angles are used larger variations in $N_\gamma$ are predicted, especially at small footing widths. It has also been reported that the factor appears to dependent on the angle of dilation, increasing as the dilation angle increases.

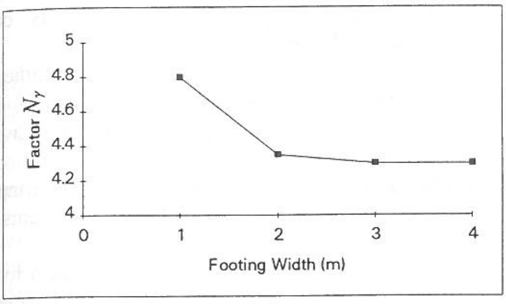


Figure 3 Effect of footing width on $N_\gamma$

However, further investigation is required to establish if this reduction is due to the increase in the number of Gauss points or the footing width.

## 4 EFFECT OF OVERALL MESH SIZE

A simple method of determining if the size of the footing governs the value of $N_\gamma$ is to multiply the overall dimensions of the mesh by the same factor. For example, if the mesh of a footing 1.5*m* wide is multiplied in both the horizontal and vertical direction by 0.1, a 0.15m footing is simulated. In this way the same mesh density and footing /mesh-dimensions ratio is kept constant. Table 1 shows the effect on $N_\gamma$ for factors of 0.1 and 10.

Table 1 Effect of mesh size on $N_\gamma$

| Multiplication Factor | Width *B* (*m*) | $N_\gamma$ |
|---|---|---|
| 0.1 | 0.15 | 4.3 |
| 1 | 1.5 | 4.3 |
| 10 | 15 | 4.3 |

Table 1 shows that using a linear-elastic perfectly plastic constitutive soil model $N_\gamma$ is not a function of the footing width and so must be influenced by the finite element mesh configuration. In the above analyses a constant footing width (*B*) to first element depth(*d*) ratio (*B/d*) was used. It is therefore necessary to establish if the depth of the first row of elements (i.e. the Gauss point depth) has an effect, especially since the shear strength of the granular soil increases with depth as the isotropic stress level increases.

## 5 EFFECT OF THE FIRST ELEMENT DEPTH

In these analyses the depth of the first row of elements shown in Figure 1 was varied to produce different *B/d* ratios. Table 2 shows typical results of the analysis for a footing width of *B*=1.5*m*.

Table 2 Effect of *B/d* ratio

| *d* (*m*) | *B/d* | $N_\gamma$ |
|---|---|---|
| 0.75 | 2 | 4.9 |
| 0.5 | 3 | 4.3 |
| 0.375 | 4 | 4.15 |
| 0.3 | 5 | 4.0 |

Table 2 shows that if $q_{ult}$ is determined by simply averaging the stresses directly under the foundation then the calculation is mesh dependent. However, the problem is that the averaged vertical stresses at the Gauss points also include the weight of the soil above and is equivalent to a foundation at the Gauss point depth (*d'*). If this approach is to be adopted then equation (5) needs to include the $N_q$ factor in the calculation. The modified or corrected value of $N_\gamma$ can then be found from equation (6) given below.

$$N_\gamma = \frac{2(q_{ult} - \gamma d' N_q)}{\gamma B} \qquad (6)$$

To determined $N_q$ the procedure given by Griffiths (1982) was used whereby all the Gauss points are set at the same initial stress value and the analysis performed as previously; $N_q$ is found by setting *B*=0 in equation (6). For an angle of friction of $\phi$=25° the finite element program computed $N_q$=10.3 for d'=0.073*m*, compared to Terzaghi's value of $N_q$=10.5 from equation (4). If the finite element value of $N_q$ is substituted back into equation (4) to determine the corrected factor $N_\gamma$ then a constant value is predicted as shown in Table 3.

Table 3 Corrected values of $N_\gamma$ for $\phi$=25°

| $d$ ($m$) | $B/d$ | Uncorrected $N_\gamma$ | Corrected $N_\gamma$ |
|---|---|---|---|
| 0.75 | 2 | 4.9 | 3.4 |
| 0.5 | 3 | 4.3 | 3.4 |
| 0.375 | 4 | 4.15 | 3.4 |
| 0.3 | 5 | 4.0 | 3.4 |

The result of $N_\gamma$ =3.4 is thought to be a more accurate value of this factor for $\phi$=25°; equations (1) & (2) predict 8.1 and 6.8 respectively. The authors have conducted similar computer simulations for other friction angles (Woodward & Griffiths 1997) and found that the $N_\gamma$ values computed in this way tend to agree with the factors predicted by Hansen & Christiensen (1969); $N_\gamma$=3.5 for $\phi$=25°. Frydman & Burd (1997) have recently calculated finite element values of $N_\gamma$ for $\phi \geq 30°$ using rather fine meshes. In these analyses nodal forces were calculated at foundation level and converted into a bearing stress. A correction factor was required however to account for the singularity at the footing edge. For smooth footings and a dilation angle of $\psi$=0° it appears that finite element values of $N_\gamma$ using a linear-elastic perfectly plastic constitutive soil model can be obtained from Table 4.

Table 4 Finite element predictions of $N_\gamma$ for smooth footings and $\psi$=0° after Woodward & Griffiths (1997) and Frydman & Burd (1997)

| $\phi$ | Woodward & Griffiths (1997) | Frydman & Burd (1997) |
|---|---|---|
| 10 | 0.3 | - |
| 15 | 0.7 | - |
| 20 | 1.6 | - |
| 25 | 3.4 | - |
| 30 | 7.3 | 7.9 |
| 35 | 17.6 | 18.9 |
| 40 | - | 42 |
| 45 | - | 92 |

## 6 CONCLUSIONS

This paper has shown that a linear-elastic perfectly plastic soil model within a finite element program does not predict a variation in $N_\gamma$ with footing width and that relatively course meshes can be used to provide a conservative estimate of its value with changing friction angle. An allowance must be made however for the surcharge component of the soil at the Gauss point depth when averaging the stresses.

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