

Featured Review: The Mathematica GuideBook for Programming. *By Michael Trott.* Springer-Verlag, New York, 2004. \$79.95. xxxvii+1028 pp., hardcover. ISBN 0-387-94282-3 (with DVD-ROM).

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About the GuideBooks. The *Mathematica GuideBooks* by Michael Trott form a four-volume set that has been in development for over a decade. The GuideBooks provide an exploration of the various facets of *Mathematica* as a tool for solving scientific problems. Each Mathematica GuideBook is available separately and covers one of the key features of *Mathematica*: programming, visualization, numerics, and symbolics. While there is some overlap and the four books build on one another, each one is self-contained and for the most part each book stands on its own. The four-volume set is over 5000 pages printed in a small font.

The author of the GuideBooks, Michael Trott, is a member of the R&D division of Wolfram Research, the makers of *Mathematica*. Readers who use The Wolfram Functions Site [7] will be familiar with Trott's work.

Leafing through the GuideBooks page by page, it is clear that Trott's mammoth work goes far beyond "The Mathematica Book" [13]. The author uses *Mathematica*'s mathematical and scientific capabilities to explore, solve, and visualize a variety of problems. The GuideBooks cover applications of *Mathematica* to classic and current problems in mathematics, computer science (mainly graphics and visualization), and some theoretical physics.

Trott's approach is as follows: First, discuss the unique features of a *Mathematica* function, including its arguments, attributes, and options. Then, give small and large examples of applications with detailed references. At the end of each chapter, Trott encourages the readers to practice what they have learned through an extensive set of exercises with solutions.

Each accompanying DVD contains the full text and executable *Mathematica* code of *all* four volumes. In addition, the DVD that accompanies a specific volume also contains the *Mathematica* output, graphics, and animations covered in that volume. Each DVD has a hyper-linked table of contents and hyper-linked index to help readers quickly find items in the entire series. The DVDs have well-designed navigation palettes, some utility notebooks and extra files. The website for the books [6] offers additional materials and updates.

Paraphrasing Trott slightly, the author hopes to accomplish the following goals: (i) Give the reader a solid working knowledge of *Mathematica*; (ii) Provide a detailed knowledge of the key aspects of *Mathematica* needed to create the "best", fastest, shortest, and most elegant solution to problems from the natural sciences; (iii) Convince the reader that working with *Mathematica* can be fruitful, enlightening, and joyful.

The GuideBooks provide a delightful tour through the world of mathematics and other scientific disciplines. No prior knowledge of *Mathematica* is needed to read the GuideBooks. In my opinion, their length and level of detail make the GuideBooks unsuitable as textbooks for courses (e.g. Modeling with *Mathematica*). Nonetheless, they are an indispensable resource for teachers, researchers, and artists alike.

This review only covers the GuideBooks for *Programming* and *Graphics*. The 1200-page *Numerics* and 1400-page *Symbolics* installments were published in late Spring 2005. Both will be reviewed at a later date.

Programming GuideBook. The novice in *Mathematica* should start with the GuideBook for Programming to learn about symbolic expressions, pure functions, rules and replacements, patterns, and list manipulations.

The Programming volume has six chapters. Chapter 1 gives a tour of *Mathematica*, showing its capabilities for numerical and symbolic computations. The chapter starts with introductory examples of *Mathematica* as an interactive system and a programming language, but rapidly moves to fairly complicated code to showcase *Mathematica's* power for graphics and visualization. On page 53 you will find one of the many striking graphs in this GuideBook. The picture shows “wrapping paper” produced with 400+ randomly placed polyhedra.

Overall, this chapter provides a sampler of the type of problems dealt with in the four GuideBooks. I enjoyed reading Section 1.3 about what *Mathematica* can and cannot do. If you are looking for creative projects for a course in, say Mathematical Modeling with *Mathematica*, you should read “What You Always Wanted To Compute.” Before you embark on one of these projects, you should consult some of the 1194 references, in addition to [1, 8, 9] for available *Mathematica* codes.

The five subsequent chapters deal with the structure of *Mathematica* expressions and with *Mathematica* as a programming language. Chapter 2 discusses the basic structure of *Mathematica's* symbolic expressions. Trott moves swiftly from elementary objects (atoms) to complicated expressions. He shows how these objects are built recursively from basic building blocks and how they can be taken apart again for analysis. Along the way, he discusses special mathematical constants, transcendental functions, branch cuts of analytic functions, and inverse functions. This is an area where paper and pencil calculations deviate from computations with *Mathematica*. Under the title “Do not be disappointed,” in Section 2.2.6, Trott explains why the standard rules for simplification (e.g. $\sqrt{a^2} = |a|$ and $\sqrt{e^x} = e^{x/2}$) are not implemented and how to avoid frustration resulting from non-simplification.

Chapter 3 offers an in-depth treatment of patterns, immediate and delayed function definitions, options and defaults, as well as attributes of *Mathematica* functions. I found this material highly technical and quite boring. Yet, there is a lot to learn from Chapter 3. For example, the distinction between delayed evaluations, as in $f[x_] := \text{something}(x)$, and immediate evaluation, i.e. $f[x_] = \text{something}(x)$, is explained well. Also, as an experienced user of *Mathematica*, I knew little about downvalues and upvalues of functions (covered in Section 3.4) and functions in the λ -calculus (Section 2.6) developed by Church and Kleene around 1940. Section 3.7 on repeated application of functions based on powerful commands such as NEST, FOLD, FIXEDPOINT is delightful reading.

In Chapter 4 with the catchy title “Meta-Mathematica,” Trott presents a collection of unrelated topics of *Mathematica* as a programming system. He covers topics such as on-line help, usage and error messages, variable protection, string manipulations, and evaluation semantics. Considerable space is devoted to the various ways of creating subprograms using the BLOCK, MODULE, and WITH commands. The design of contexts and packages are also covered in detail.

Knowing how to apply the emergency brake (Section 4.2.2) and how to limit time and memory resources are important because, from time to time, we all “master the art” of writing infinite loops! In Section 4.4, communication with the outside world—to read and write *Mathematica* files—is discussed, which gives me the opportunity to warn you about two “undocumented features” of *Mathematica* under Microsoft Windows. First, do not use `<<"filename"` to read files, instead use `Get["filename"]` for the latter command line does not get corrupted. Second, Trott’s GuideBooks give the impression that it is preferable to work within notebooks. In my lectures on *Mathematica*, I stress that this is a recipe for disaster. One should always keep data and executable code (e.g. a text file with the *Mathematica* program) outside the notebook and read them in when needed. Why? Sooner or later and without your wrong-doing, part of your (long) *Mathematica* notebook may mysteriously turn into a black box. When that happens, your work will disappear in the abyss along with your corrupted notebook.

The main topics of Chapter 5 are replacement rules and patterns. Both are essential for advanced programming. Trott also reviews logical operations, control structures (IF, FOR, WHILE, WHICH, SWITCH), and Boolean expressions. Although highly technical, the use of pattern matching and conditional arguments are at the heart of symbolic computation. In this chapter Trott gives a large set of examples showing how the powerful paradigm of rule-based programming can be used for short and elegant solutions of a variety of problems. A nice example relates to graph theory where he draws the House of Nikolaus (page 635).

Chapter 6 brings us back to more familiar terrain. It discusses manipulations of data sets, vectors, and matrices. Operations for vector calculus are covered in Section 6.4.3. Next comes linear algebra (Sections 6.5.1 and 6.5.3). For manipulations with tensors the reader is referred to [2, 3]. All are dealt with in the larger context of manipulations of lists, because that is how vectors, matrices, and tensors are represented in *Mathematica*. As amply shown, the possibility of manipulating lists as whole entities leads to concise and effective programs. Trott walks us through ways of creating lists, formatting and visualizing lists, and manipulations of single and nested lists. The two most surprising examples of list manipulations (to me) involve making crossword puzzles and creating and solving magic squares in Sections 6.4.4 and 6.5.2, respectively. Two nice examples from physics, involving Lorentz transformation and quantum cellular automata, are given in Section 6.5.1.

I found Section 6.6 quite interesting. There, Trott investigates the frequency of use of the 2000+ built-in *Mathematica* functions to distill the top ten most popular ones. It comes as no surprise that the functions LIST, PATTERN, and BLANK win gold, silver and bronze!

On the whole, the Programming GuideBook provides a comprehensive, step-by-step development of *Mathematica* programming capabilities and contains an impressive collection of examples and worked exercises. Key *Mathematica* functions are discussed in detail, used in interesting examples, and put to the test in real programs. Through experimentation with the DVD, you will become fluent in working with *Mathematica* expressions, pure and named functions, and gain experience in procedural, rule-based, and functional programming.

Graphics GuideBook. Through an abundance of examples, this volume teaches the reader how to use *Mathematica* to visualize functions and data, manipulate graphics, and optimize their appearance.

There are three long chapters in this volume. The first two chapters deal with 2D and 3D graphics. There you learn how to create pretty images from graphics primitives, such as points, lines, and polygons. Little attention is paid to computational geometry and texturing of surfaces. Chapter 3 is devoted to contour and density plots. Let me now briefly review the various chapters.

Chapter 1 (which spans 430 pages) starts by examining 2D graphics primitives and their options. Section 1.1 on fundamentals is far from fundamental. It contains creative one-liners and sophisticated code for complex Cantor sets, visualizations of the Pythagorean theorem, pine cones, Julia sets, and fractal graphs.

Section 1.2 covers plots of functions based on analytic formulas (including parametric representations), as well as functions given in discrete form or defined through data sets. Section 1.3 shows how to combine and animate images. Again it is clear that Trott's forté is graphics. The example with the mice from a German TV show (pages 194-199) illustrates his true talent.

One of the most delightful sections is Section 1.5 on iterative graphics, with the Sierpinski triangle, Peano's curves, Penrose tiling, Barnsley's fern, Lindenmayer's grasses and herbs, and Koch curves, as leading examples. Some of these images appear in other *Mathematica* books and in The Mathematica Graphics Gallery [5].

Subsection 1.5.8 on Escher drawings is eye-catching. Section 1.6 on "coloring curves" misses its punch in a book that is printed in black-and-white. Here again the DVD comes to the rescue. A non-exhausted reader can consult the 839 references at the end of Chapter 1.

Despite the stunning graphs, Chapter 1 comes across as an encyclopedia of aesthetically pleasing pictures, interlaced with boxed function descriptions, and occasional highlighted remarks or warnings.

The story repeats in Chapter 2, except that this chapter surveys 3D graphics. I leafed through the 280 pages and briefly paused at the pictures of diamonds, candelabras, Sierpinski sponges, trefoil knots, Easter eggs, Klein bottles, etc. Section 2.4 deals with the construction and visualization of 3D Brillouin zones, a family of complicated polyhedra of relevance to physics and material science. The author forewarns that some of the codes in Chapter 2 are not optimized with respect to efficiency and length and may run for a long time. Chapter 2 concludes with 20 exercises (with solutions) and 540 references, the latter partly overlap with those in Chapter 1.

The 120-page Chapter 3 covers contour and density plots. Again, the reader is faced with a plethora of kaleidoscopic pictures of, for example, branch cuts of analytic functions, plane wave superpositions, Goffinet dragons, and so on. Because of the practical importance of equipotential surfaces, a large section is devoted to contour plots in 3D.

All in all, the Graphics GuideBook confronts you with a huge collection of 2D graphics, contour plots, plots of surfaces, free-form 3D surfaces, and animations. Hundreds of detailed examples and programs, from small to large, illustrate visualization techniques, methods and algorithms. With the tools in this book one can create virtually any possible graphic, from mathematical curves and surfaces to artistic images.

Strengths and Weaknesses. The GuideBooks have an overwhelming amount of information with a fair amount of repetition. The material is a mix of function descriptions, techniques, notes, code samples, graphics, mathematics, and physics. It takes some skill to get the most out of these titanic GuideBooks. For example, paging through the books, I did not always see where one item ended and another began. Shorter, better organized chapters with clearly divided sections would have been preferable.

Differing from other books about *Mathematica*, Trott prevents a dry exposition of the minutiae of coding by weaving in historical tidbits about mathematics and physics, and adding clever programming tricks. His style is informal and the mathematical background material is often sketchy. As a mathematician, I had hoped to read more without having to consult the references (which should have been listed in proper numerical order).

Trott's compendium is a printout of the *Mathematica* notebooks on the DVDs. As such, the printed version could have benefited from professional editing. A more polished presentation, snappier language, shorter sentences, and less repetition, would have helped. In short, quantity got the better of quality.

Moving from section to section, the sample codes (and graphics) get more complicated. An expert in *Mathematica* can learn a lot from these examples. A newcomer, however, will have to take the codes apart to understand the details. I would have preferred a few carefully selected examples with lucid descriptions of how the codes were put together in the first place.

On the positive side, the breadth of covered topics is staggering. Trott gives many original examples, some inspired by research (with proper citation); others based on news groups threads. The GuideBooks include solutions to the 1,000+ exercises, which are tagged according to level of difficulty. The solutions take up hundreds of pages in the books.

In many ways, the DVD version of the GuideBooks is superior to the printed version. In addition to the color graphics, hyperlinking, navigation palettes, and animations, you can immediately experiment with and build upon Trott's code.

Conclusions. The main goals of the GuideBooks were to demonstrate, showcase, teach, and exemplify scientific problem solving with *Mathematica*. Trott certainly accomplished these goals. Over the past fifteen years *Mathematica* has grown from "a system for doing mathematics by computer" into "a system for technical computing." Likewise, the GuideBooks are centered around *doing* mathematics and the electronic version of the GuideBooks do indeed illustrate that *Mathematica* is a superb tool for computer-assisted mathematics and visualization.

The multi-platform DVD that comes with each book is a terrific asset for whoever foregoes buying the complete set. Indeed, each DVD contains the fourteen large *Mathematica* notebooks each of which corresponds to a chapter in the four printed books. Thus, buying one of the GuideBooks gives you access on DVD to the text and executable codes from all four GuideBooks.

Let me conclude by mentioning two additional treasures: (i) MathWorld [11], which is now part of ScienceWorld [12] by Eric Weisstein [10], who designs free interactive encyclopedias for mathematics, physics, chemistry, and astronomy with *Mathematica*; (ii) The Wolfram Functions Site [7], largely developed by Michael Trott. This public-service site features 80,000+ formulas and identities, and thousands of visualizations. In the same spirit, Trott's GuideBooks (at an economical

price of \$79.95 each) are a gift to the scientific community. Certainly, I see The GuideBooks become an invaluable, lasting resource for aficionados of *Mathematica*. Whether you plan to use the GuideBooks for teaching, research, or recreational purposes, Trott's passion for *Mathematica* is contagious. I encourage you to go play with the *Mathematica* "tools and toys" in the GuideBooks!

References

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