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THE NTH ORDER APPROXIMATION METHOD IN ACOUSTO–OPTICS AND THE CONDITION FOR "PURE" BRAGG REFLECTION

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INTRODUCTION

It is well–known that **Bragg diffraction** in acousto–optics occurs if the incident light makes a Bragg angle with the ultrasonic wave planes and the diffraction spectrum only consists of the orders –1, 0 and +1.

"Pure" Bragg reflection arises if the diffraction results in a spectrum of orders 0 and +1, with evanescent order –1 (Figure 1). Theoretically those phenomena were respectively described by Nagabhushana Rao [1] and Phariseau [2], approximating the Raman–Nath system for the amplitudes of the diffracted light waves. Those results may be rederived from the NOA method [3] for $N = 1$ and from the MNOA method [4] for $M = 0$, $N = 1$ and treated as eigenvalue problems. We shall compare both solutions with the experimental data of Klein et al. [5]. Further we employ the 1OA method to investigate the occurrence of "pure" Bragg reflection for large and increasing values of the Klein–Cook parameter Q and the Raman–Nath regime parameter ρ.

THE NOA AND MNOA METHODS

The starting point for those methods is the Raman–Nath system for the amplitudes of the diffracted light waves [6]

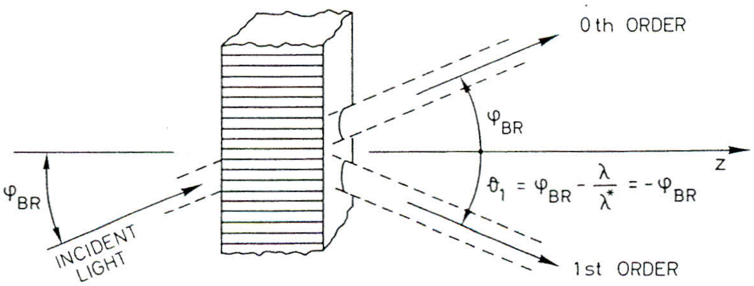

Figure 1. Geometry of single–order Bragg diffraction ("pure" Bragg reflection).

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$$2\frac{d\phi_n}{d\zeta} - (\phi_{n-1} - \phi_{n+1}) = i\rho n(n + \beta)\phi_n, \tag{1}$$

with boundary conditions

$$\phi_n(0) = \delta_{n0} \quad , \quad n = 0, \pm 1, \pm 2, \ldots \tag{2}$$

In (1), $\zeta = \pi\varepsilon_1 z/\varepsilon_r\lambda\cos\varphi$, $\rho = 2\varepsilon_r\lambda^2/\varepsilon_1\lambda^{*2}$, $\beta = -(2\lambda^*/\lambda)\sin\varphi$, where φ is the angle of incidence of the light (the z-axis being parallel to the ultrasonic wave fronts), ε_1 the maximum variation of the relative permittivity ε_r, λ the wave length of the light in the medium, λ^* the wave length of ultrasound. If $\beta = -p$, with p integer, then $p(\lambda/2\lambda^*) = \sin\varphi_{BR}^{(p)}$, where $\varphi_{BR}^{(p)}$ is called the Bragg angle of order p. We also introduce the Raman–Nath (RN) parameter $v = \zeta L/z$, L being the width of the ultrasonic field, and the Klein–Cook parameter $Q = v\rho$. In the NOA method [3] one neglects the energy in the diffraction orders higher than N and lower than –N. The truncated system can then be solved by an eigenvalue method. For $N = 1$ we obtain for $\beta = -1$ ($\varphi = \varphi_{BR}^{(1)}$)

$$I_{-1} = 4\,[s_1 s_2 S_1 \sin^2(s_1 - s_2)\,\tfrac{v}{4} + \text{cycl.}], \tag{3}$$

$$I_0 = 1 + 4\,[s_1 s_2 S_1 (2\rho - s_1)(2\rho - s_2)\sin^2(s_1 - s_2)\,\tfrac{v}{4} + \text{cycl.}], \tag{4}$$

$$I_{+1} = 4\,[S_1(2\rho - s_1)(2\rho - s_2)\sin^2(s_1 - s_2)\,\tfrac{v}{4} + \text{cycl.}], \tag{5}$$

with

$$S_1 = 1/(s_1 - s_2)^2(s_1 - s_3)(s_2 - s_3), \tag{6}$$

and where s_1, s_2, s_3 are the single real roots of the characteristic equation

$$s^3 - 2\rho s^2 - 2s + 2\rho = 0. \tag{7}$$

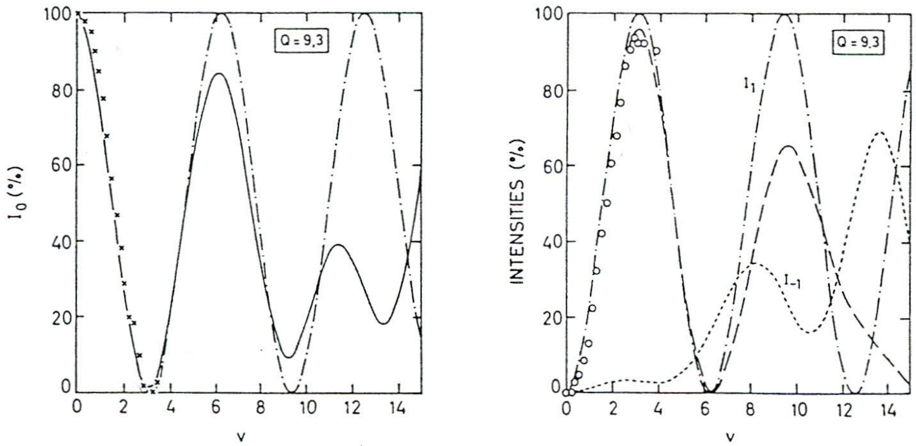

Figure 2. I_0 versus v (left) for $Q = 9.3$ and $\beta = -1$ calculated from Phariseau's formula (8) (–·–·–) and from Nagabhushana Rao's formula (4) (——) compared with experimental data of Klein et al (×××).
I_1 versus v (right) calculated from Equation (9) (–·–·–) and from (5) (– – –) compared with experimental results of Klein et al (o o o) and I_{-1} versus v from (3) for the same values of Q and β.

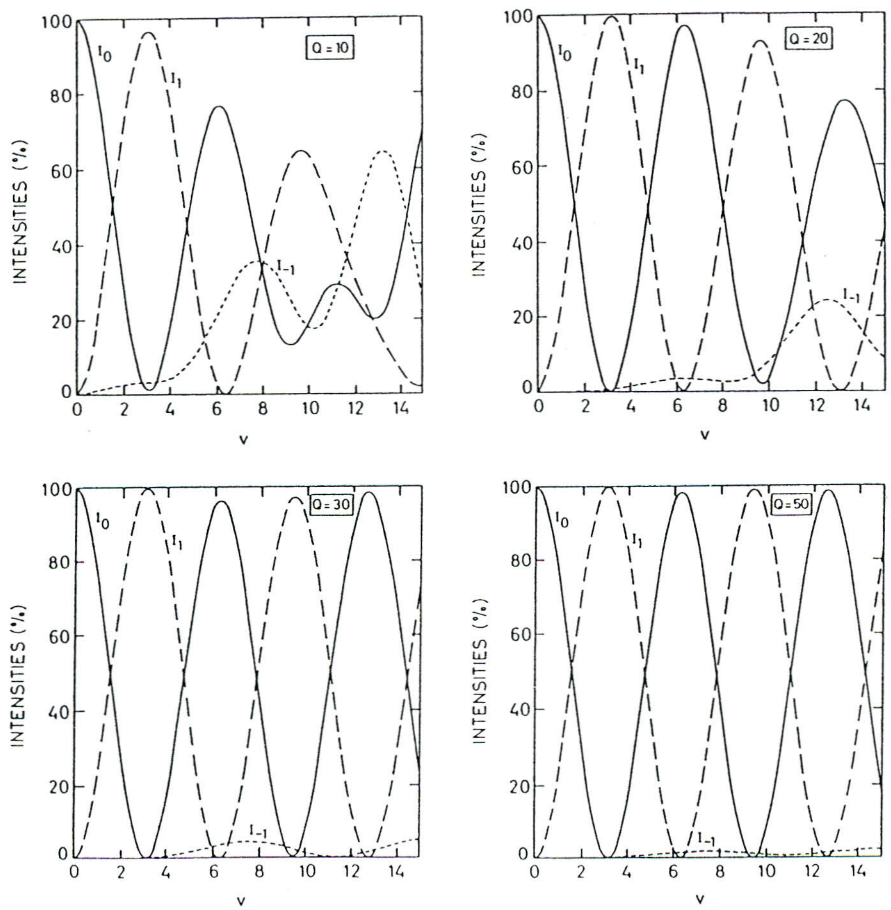

Figure 3. I_0(———), I_1(— — —) and I_{-1}(- - -) versus v at Bragg incidence (β = −1) respectively calculated from (4), (5), (3) for Q = 10, 20, 30, 50.

Those results are in fact Nagabhushana Rao's formulae [1] written in a more explicit form, for $\beta = -1$. In the MNOA method [4] it is assumed that only M negative and N positive orders are present in the diffraction spectrum, with $M \leq N$ for $\varphi > 0$. Considering $M = 0$, $N = 1$ and using the eigenvalue method we obtain, for perfect Bragg diffraction $\varphi = \varphi_{BR}^{(1)}$ $(\beta = -1)$, Phariseau's well–known results [2],

$$I_0 = \cos^2(v/2) \tag{8}$$

$$I_1 = \sin^2(v/2). \tag{9}$$

NUMERICAL RESULTS

In Figure 2 the intensities I_0 (left) and I_1 and I_{-1} (right) versus v are shown. The various curves are calculated with formulae (8,9) and with (3), (4), (5). Both sets of theoretical results are compared for Q = 9.3 with experimental data obtained by Klein et al. [5]. The fitting of the 1OA curves with the experimental points is excellent. Unfortunately the data are restricted to the domain $v \in [0,4]$. In this region for v there is a rather good agreement with Phariseau's results, but it fails beyond $v \approx 5$, due to the fact, that from thereon I_{-1} is no longer negligible. Hence, we can conclude that for $Q = 9.3$ there is only "pure" Bragg reflection up to $v \approx 5$.

In Figure 3 we represent the curves for I_0, I_{+1} and I_{-1} versus v, calculated with the 1OA method at Bragg incidence (Equations (3), (4), (5)), for increasing values of the Klein–Cook parameter, namely $Q = 10, 20, 30, 50$. The larger the value of Q, the better the condition for "pure" Bragg reflection is satisfied. This is because the intensities I_{-1} decrease with higher values of v. Incidentally, the deviation of the curves for I_0 and I_1 from Phariseau's theory becomes small with larger Q.

Finally, in Figure 4 we show I_0, I_{+1} and I_{-1} versus v, computed from Equations (3), (4) and (5) at Bragg incidence, but now for increasing values of the regime parameter, i.e. $\rho = 1, 5, 10$ and 20. Similar calculations were performed for $\rho = 30, 40$ and 50, but the results were identical with those for $\rho = 20$. Observe that for $\rho = 1$ most values of I_{-1} are too large, and second order intensities are not negligible, so that this case does not illustrate Bragg reflection very well. But for $\rho \geq 5$, the calculated values of I_{-1} keep decreasing, practically vanishing for $\rho = 20$. Furthermore, I_0 and I_{+1} are nearly represented by Phariseau's formulae (8) and (9). This shows that approximately for $5 \leq \rho \leq 20$ there is near Bragg reflection, whereas for $\rho \geq 20$ we have

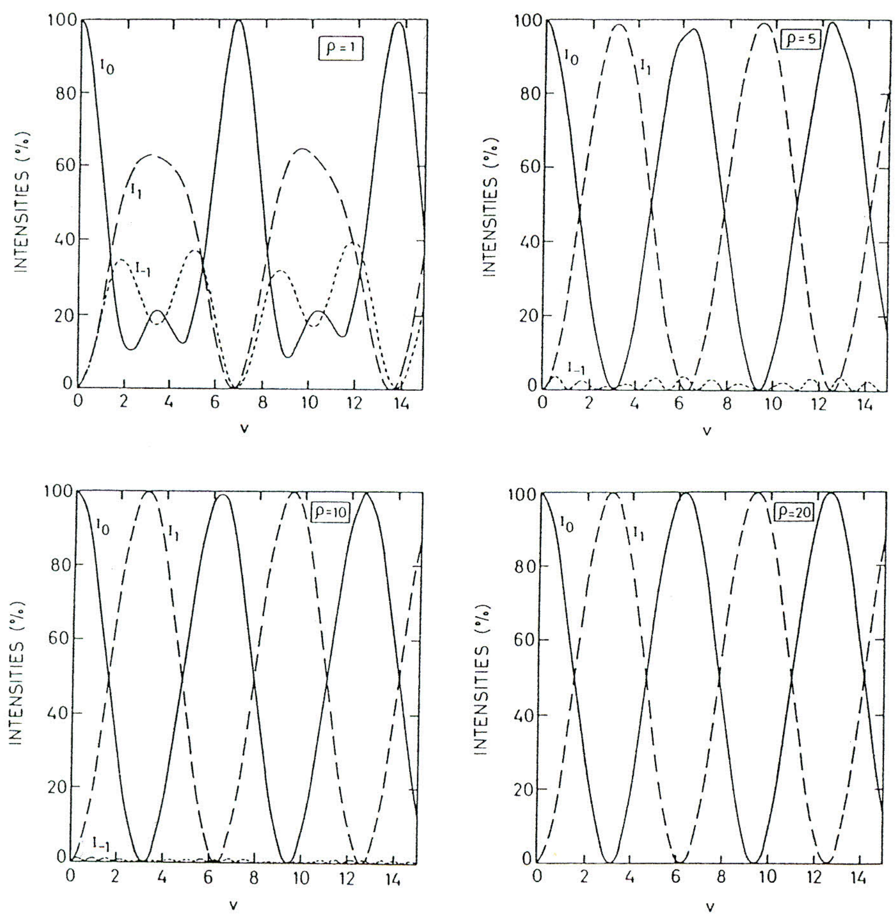

Figure 4. I_0(———), I_1(— — —) and I_{-1}(- - -) at Bragg incidence ($\beta = -1$) calculated from (4), (5), (3) for $\rho = 1, 5, 10$ and 20.

"pure" Bragg reflection. Hence, it is clear that both the parameters Q and ρ are relevant for determining the condition for Bragg reflection.

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