Interpolation and Spatial Transformations

Examples
Example 1

- We want to find the transformation from the left image below to the right image (you can find these images on the course website).

- Calculate the affine transform matrix \( T \) that will map \((u,v)\) points in the input (left) image to \((x,y)\) points in the output (right) image according to the equation
  \[
  \begin{bmatrix}
  x \\ y \\ 1
  \end{bmatrix} = \begin{bmatrix}
  u \\ v \\ 1
  \end{bmatrix} T
  \]

- What coordinates does the point \((u,v) = (82,18)\) map to in the output image?

- What coordinates does the point \((x,y) = (30,55)\) map to in the input image?

(It is not necessary to find corresponding points automatically, you can find them by hand, using say, “imtool” or “imshow” with “impixelinfo”.)
Example 1
Example 1
Example 2

• Adapt the Matlab code on slide 6 of lecture 4, to perform a general transformation with a matrix T.

• Apply it to the cameraman image to do each of these transformations:
  – Translate to the right by 30 pixels and down 20 pixels.
  – Rotate the image clockwise by 45 degrees.
  – Expand by a factor of 2.0 horizontally and shrink by a factor of two vertically.
Example 3

• Using matrix equations, calculate the affine transform from the image on the left to register (align) it to the image on the right.
Example 3 (continued)

• We find corresponding points by hand
• We calculate T using these equations

\[
\begin{bmatrix}
    x_1 & y_1 & 1 \\
    x_2 & y_2 & 1 \\
    x_3 & y_3 & 1 \\
    \vdots & \vdots & \vdots \\
    x_n & y_n & 1 \\
\end{bmatrix}
= \begin{bmatrix}
    u_1 & v_1 & 1 \\
    u_2 & v_2 & 1 \\
    u_3 & v_3 & 1 \\
    \vdots & \vdots & \vdots \\
    u_n & v_n & 1 \\
\end{bmatrix} T
\]

\[
X = UT \\
U^T X = U^T UT \\
(U^T U)^{-1} U^T X = T
\]

• Apply the transformation to the image.
Example 3 (continued)
Example 3 (continued)
Example 4 - Image resolution study

FIGURE 2.22  (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

• Download images from course website
  – Fig0222(a)(face).tif
  – Fig0222(c)(crowd).tif
  – (cameraman.tif is already available in Matlab)
Example 4 (continued)

• Experiment with N and k
  – Matlab’s “imresize” will resize the image to whatever size you want
  – To specify k, just scale the image to between 0 and $2^k - 1$

```matlab
N = 128;
k = 4.67;
I1 = imresize(I, [N,N]) * ((2^k-1)/255);
imshow(I1, []);
title(sprintf('N = %d, k = %f', N, k));
```

puts a title string over the image

sprintf works just like in C ... it prints these values to a string
Example 4 (continued)

- See if you get (approximately) the same isopreference curves as this study:
  - Generate an image with a particular value of N and k
  - Then generate it with a different value of N and k
  - Adjust N and k for the second image until the subjective quality is about the same as the first
Example 4 (continued)

clear all
close all

I = imread('Fig0222(a)(face).tif');
% I = imread('cameraman.tif');
% I = imread('Fig0222(c)(crowd).tif');

% Display first (reference) image.
N = 128;
k = 4.67;
I1 = imresize(I, [N,N]) * ((2^k-1)/255);
subplot(1,2,1), imshow(I1, []);
title(sprintf('N = %d, k = %f', N, k));

% Compare to second image (hit control-C to exit).
while true
    N = input('N = ');
k = input('k = ');

    I2 = imresize(I, [N,N]) * ((2^k-1)/255);
    subplot(1,2,2), imshow(I2, []);
    title(sprintf('N = %d, k = %f', N, k));
end