Colorado School of Mines

Image and Multidimensional Signal Processing

Professor William Hoff
Dept of Electrical Engineering & Computer Science
http://inside.mines.edu/~whoff/
Intensity Transformations
Gray Level Transformations - Examples

- Map an input value $r$ to an output value $s$
- Examples:
  - Contrast stretching
  - Thresholding

![Diagram showing gray level transformations](image)

- Most image editing software (e.g., Adobe Photoshop) allows you to do transformations like this
Example – gamma transform (exponential)

- Each input value is raised to the power of gamma
- Gamma transforms were often used to correct the intensities in CRT displays

**FIGURE 3.6** Plots of the equation $s = cr^\gamma$ for various values of $\gamma$ ($c = 1$ in all cases).
Matlab examples

• Image negative
  – \( I = \text{imread}('rice.png'); \)
  – \( I_n = 255 - I; \)  \% image negative

• Gamma transform
  – \( I = \text{imread}('Fig0308(a).tif'); \)  \% image of spine
  – \( I = \text{double}(I)/255; \)  \% scale to 0..1 (could also use \text{im2double})
  – \( I_g = \text{power}(I, 0.3); \)  \% enhances dark or light?

  – \( I = \text{imread}('Fig0309(a).tif'); \)  \% aerial image
  – \( I = \text{double}(I)/255; \)  \% scale to 0..1
  – \( I_g = \text{power}(I, 4.0); \)  \% enhances dark or light?
Quick Review of Probability Concepts (2.6.8)

• Probability
  – We do an experiment (e.g., flip a coin) $N$ times
  – We count number of outcomes of a certain type (e.g. heads)
  – Probability of getting that outcome is the relative frequency as $N$ grows large
    \[
    P(\text{heads}) = \frac{n_H}{N}
    \]
  – Probability of a particular outcome is between 0 and 1
  – Probabilities of all outcomes sum to 1

• Random variable
  – Takes on values as a result of performing an experiment (i.e., maps experimental outcomes to real numbers)
  – Example
    • Random variable $x$ represents coin toss
    • E.g., $x=0$ for heads and $x=1$ for tails
    • $P(x=0) = 0.5$, $P(x=1) = 0.5$

For more help see http://www.imageprocessingplace.com/root_files_V3/tutorials.htm
Quick Review of Probability Concepts (cont)

• Mean (or expected value) of a random variable

\[ \bar{x} = \mu_x = E[x] = \frac{1}{N} \sum_{i=1}^{N} x_i \]

  – where \( x_i \) is the value of the \( i \)th experiment

• Say \( x \) can take on the values \( r_0, r_1, \ldots r_{L-1} \)

• Then

\[ p(x=r_k) = \frac{n_k}{N} \]

  – where \( n_k = \) number of times \( r_k \) occurs in \( N \) trials

• The mean is

\[ \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{1}{N} \left[ n_0 r_0 + n_1 r_1 + \ldots + n_{L-1} r_{L-1} \right] \]

\[ = \left[ \frac{n_0}{N} r_0 + \frac{n_1}{N} r_1 + \ldots + \frac{n_{L-1}}{N} r_{L-1} \right] = \sum_{k=0}^{L-1} p(r_k) r_k \]
Quick Review of Probability Concepts (cont)

- Variance of a random variable is
  \[
  \sigma^2 = E\left[ (x - \mu_x)^2 \right] = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_x)^2
  \]

- Also
  \[
  \sigma^2 = \sum_{k=0}^{L-1} p(r)(r_k - \mu_x)^2
  \]

- Note - sometimes it is easier to compute variance using
  \[
  \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_x)^2 = \frac{1}{N} \sum_{i=1}^{N} \left( x_i^2 - 2x_i\mu_x + \mu_x^2 \right)
  \]
  \[
  = \frac{1}{N} \sum_{i=1}^{N} x_i^2 - \frac{1}{N} \sum_{i=1}^{N} (2x_i\mu_x) + \frac{1}{N} \sum_{i=1}^{N} \mu_x^2 = E[x^2] - \mu_x^2
  \]
Quick Review of Probability Concepts (cont)

- *Continuous* random variables (as opposed to *discrete*) can take on non-integer values (e.g., temperature)

- We can’t talk about the probability of \( x \) taking a specific value, but we can give the probability of \( x \) having a value somewhere in a range

- The cumulative probability distribution function (CDF) is
  \[
  F(a) = P(-\infty < x \leq a) \quad 0 \leq F(x) \leq 1
  \]

- The probability density function (pdf) is
  \[
  p(x) = \frac{dF(x)}{dx}
  \]

- The probability that \( x \) is between \( a \) and \( b \) is
  \[
  P(a < x \leq b) = F(b) - F(a) = \int_{a}^{b} p(w) \, dw
  \]
Image Histograms

- Histogram is a count of the number of pixels $n_k$ with each gray level $r_k$
  \[ n_k = h(r_k) \]
- It is an approximation of the probability density function
  \[ p(r_k) = \frac{n_k}{N} \]
Example

- Find histogram
- Find pdf, CDF
- Find mean
- Find variance
Histogram Equalization

• We can transform image values to improve the contrast

• Want histogram of the image to be flat

• This will make full use of the entire display range

• This is called histogram equalization
Histogram Equalization

• Let the histogram of the input image be \( H(r) \)

• The pdf of the input image is
  \[
  p_r(r) = \frac{H(r)}{N}
  \]

• We want a transformation \( s = T(r) \) that will give an output image whose histogram is flat:
  \[
  p_s(s) = \text{const}
  \]

• The transformation should be a monotonically increasing function
  – this prevents artifacts created by reversals of intensity
Histogram equalization (contd.)

- Consider the cumulative probability distribution function of the input image

\[ F(r) = \int_{0}^{r} p_r(w) \, dw \]

- If we use this as our transformation function (scaled by the maximum value \( L-1 \)), the output image will have \( p_s(s) = \text{const} \)

\[ s = T(r) = (L-1) \int_{0}^{r} p_r(w) \, dw \]
Histogram Equalization

- (Show this from considerations of probability)
Histogram Equalization

- Histogram equalization – Matlab’s `histeq`
  - try “liftingbody.png”
Doing histogram equalization by hand

- Get histogram of $M \times N$ input image $H_i(r) = n_r$. Gray levels range from $0..L-1$.
- Determine probability density function (pdf)
  \[ p_r(r_k) = \frac{n_k}{MN} \]
- Determine cumulative probability distribution (CDF)
  \[ F_r(r_k) = \sum_{j=0}^{k} p_r(r_j) \]
- Scale $T(r)$ to desired range of output gray levels
  \[ T(r) = (L-1) F_r(r) \]
- Apply the transformation $s = T(r)$ to compute the output values
Example

- 64x64 image
  - \( M \times N = 64 \times 64 = 4096 \)
- 3 bits/pixel
  - Gray levels range from 0 to \( L-1 \)
  - \( L = 2^3 = 8 \)

<table>
<thead>
<tr>
<th>( r_k )</th>
<th>( n_k )</th>
<th>( p_r(r_k) = n_k/MN )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_0 = 0 )</td>
<td>790</td>
<td>0.19</td>
</tr>
<tr>
<td>( r_1 = 1 )</td>
<td>1023</td>
<td>0.25</td>
</tr>
<tr>
<td>( r_2 = 2 )</td>
<td>850</td>
<td>0.21</td>
</tr>
<tr>
<td>( r_3 = 3 )</td>
<td>656</td>
<td>0.16</td>
</tr>
<tr>
<td>( r_4 = 4 )</td>
<td>329</td>
<td>0.08</td>
</tr>
<tr>
<td>( r_5 = 5 )</td>
<td>245</td>
<td>0.06</td>
</tr>
<tr>
<td>( r_6 = 6 )</td>
<td>122</td>
<td>0.03</td>
</tr>
<tr>
<td>( r_7 = 7 )</td>
<td>81</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Example 3.5 in book
Example (continued)

- Excel spreadsheet

<table>
<thead>
<tr>
<th>r</th>
<th>H[r]</th>
<th>p[r]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>790</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1023</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>850</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>656</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>329</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>245</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4096</td>
<td></td>
</tr>
</tbody>
</table>
Example (continued)

• To calculate histogram of transformed image $H_s(s)$:
  - For each value of $s$
    • Find values of $r$ where $s = T(r)$
    • Sum $H_r(r)$ for those values

• Example:
  - Take $s=6$
  - $T(r) = 6$ for $r=3,4$
  - $H_s(6) = H_r(3) + H_r(4)$
    = $656 + 329 = 985$

• Excel spreadsheet
  - Use “SUMIF” formula
    = SUMIF( range-to-test, cell-to-check-if-equal, cells-to-sum)
Manual Histogram Equalization - Example

<table>
<thead>
<tr>
<th>r</th>
<th>H(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>
Adaptive (Local) Histogram Equalization

• Divide image into rectangular subregions (or “tiles”), do histogram equalization on each

• To avoid “blocky” appearance:
  – Make tiles overlapping
  – Or, interpolate across tiles

• Matlab’s `adapthisteq`
  – Optional parameters
    • ‘NumTiles’ *default is [8 8]*
    • ‘ClipLimit’ (0..1; limits # pixels in a bin; higher numbers => more contrast) *default is 0.01*

• Try “liftingbody.png”
Matlab example

• Read image

\[ I = \text{imread('liftingbody.png')}; \]

• Do regular histogram equalization and adaptive histogram equalization

\[ I_{eq} = \text{histeq}(I); \]
\[ I_{adapteq} = \text{adapthisteq}(I); \]

• Display results
  – “subplot” allows you to put multiple images in a single figure

\[ \text{subplot(1,3,1), imshow(I,[]); % row, cols, index} \]
\[ \text{subplot(1,3,2), imshow(I_{eq},[]); % row, cols, index} \]
\[ \text{subplot(1,3,3), imshow(I_{adapteq},[]); % row, cols, index} \]
Summary / Questions

• Gray level transformations map each input intensity value to an output intensity value.

• We can use these transformations to improve the contrast in an image.

• Why is it desirable to have a flat histogram in the output image?