Spatial Filtering

Examples
Example 1 – successive applications

• Consider if you convolved a filter to an image twice in a row. This is effectively the same as convolving a larger filter with the image once.

• To see this, recall that convolution is associative

\[ f * (g * h) = (f * g) * h \]

• By hand, convolve a 3x3 box filter (containing all 1s) with itself (padding with zeros). What is the effective larger filter?

\[
\begin{array}{ccc}
+1 & +1 & +1 \\
+1 & +1 & +1 \\
+1 & +1 & +1 \\
\end{array}
\]
Example 1
Example 2 (Matlab)

• Repeat example 1 using Matlab: Use Matlab’s “conv2” to convolve the 3x3 “box filter” operator with itself

\[
\begin{align*}
  &\text{>> } B = \text{ones}(3,3) \\
  &\text{>> } B2 = \text{conv2}(B,B)
\end{align*}
\]

• Show that on a simple image, doing \(B^* (B^* I)\) is the same as \((B^* B)^* I\)

\[
\begin{align*}
  &\text{>> } I = \text{rand}(5,5) \\
  &\text{>> } \text{conv2(} \text{conv2(I,B),B} \text{)} \\
  &\text{>> } \text{conv2(I,B2)}
\end{align*}
\]
Example 3 – Prewitt and box

- Find the effective filter if you convolve an image with a 3x3 box filter \( B \) (all ones), then convolve with the Prewitt filter \( P \).

\[
\begin{align*}
\text{>> } & B = \text{ones}(3) \\
& B = \\
& \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\
\text{>> } & P = [-1 0 1; -1 0 1; -1 0 1] \\
& P = \\
& \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \\
\text{>> } & \text{conv2}(P,B)
\end{align*}
\]

- Is this the same as if you did it in reverse order?
Example 4 – 3rd derivative

• What would a discrete filter look like, that performs a 3\textsuperscript{rd} derivative?

• Recall
  – First derivative
    \[
    \frac{\partial f(x, y)}{\partial x} \approx f(x+1, y) - f(x, y)
    \]

  – Second derivative
    \[
    \frac{\partial^2 f(x, y)}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial f(x, y)}{\partial x} \right) = f(x+1, y) - 2f(x, y) + f(x-1, y)
    \]
Example 4 – 3rd derivative (continued)

• 3rd derivative
Example 5 – separable filter

• A 2D convolution or correlation can be implemented more efficiently if the filter \( w(x,y) \) is separable, meaning that it can be written as a product of two functions, one that is a function only of \( x \) and the other only of \( y \). In other words, \( w(x,y) = w_x(x) \cdot w_y(y) \).

• Prove that a 2D correlation with a separable filter can be computed by (1) computing a 1D correlation with \( w_y(y) \) along the individual columns of the input image, followed by (2) computing a 1D correlation with \( w_x(x) \) along the rows of the result from (1).

\[
w(x, y) \ast f(x, y) = w_x(x) \ast \left[ w_y(y) \ast f(x, y) \right]
\]
Example 5 – separable filter (continued)

• The 2D correlation operation is

\[ w(x, y) \ast f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t) \]

• Substitute \( w(s,t) = w_x(s) w_y(t) \):

\[ : \]
Example 6 – box separable filter

• The 3x3 box filter is separable.
\[ w(x, y) = w_x(x)w_y(y) \]

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
+1 & +1 & +1 \\
+1 \\
+1 \\
\end{array}
\]

• Show on a simple image that the 2D correlation is the same as doing the two 1D correlations. Use this as the image.

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
Example 6 – box separable filter (continued)

- 2D correlation \( w_{xy}(x,y) * I \)
  
  \[
  \text{filter2}(wxy,I, 'full')
  \]

- Two successive 1D correlations \( w_y(y) * (w_y(y) * I) \)
  
  \[
  \text{filter2}(wy, \text{filter2}(wx,I, 'full'), 'full')
  \]
Example 7

• On the course website there are two images, I210.tif and I250.tif, which are from a video of a street corner.

• Extract a subimage template of the white car from the first image (you can use Matlab’s “imcrop” function).

• Use the method of normalized cross-correlation to match this template to the second image (you can use Matlab’s “normxcorr2” function).

• The maximum correlation score should occur at the center of the white car in the second image – mark that point with a rectangle (or some other type of marker overlay) on the second image.