Fourier Transform

Examples
Example 1

• Find complex conjugate of $a = 1 - 2j$

• Find magnitude of $b = -4 + 5j$

• Find product of $a*b$
Example 2

- Find polar representation of -2 – 3j

- Find Cartesian representation of $5e^{-j3}$

- Try the Matlab functions to compute these
Example 3

• Find the integration of the continuous impulse function with
  \[ \int_{-\infty}^{\infty} (2t^2 + 1)\delta(t)\,dt \]
  \[ \int_{-\infty}^{\infty} \sin(2\pi t)\delta(t - 1)\,dt \]

• Find the summation of the discrete impulse function with
  \[ \sum_{x=-\infty}^{\infty} 3e^{-\pi x} \delta(x) \]
  \[ \sum_{x=-\infty}^{\infty} \cos(2\pi x + \pi / 2)\delta(x + 1/2) \]
Example 4

- Find the Fourier transform of the one-sided “box” or “rectangle” function

\[ f(t) = \begin{cases} 
A & 0 \leq t \leq T \\
0 & \text{otherwise} 
\end{cases} \]
Example 5

- In Matlab, create a series of numbers \( f(0), f(1), ..., f(M-1) \).
- Find the Discrete Fourier Transform (DFT) of the series, directly from the definition:

\[
F(u) = \sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \quad \text{for } u = 0, 1, ..., M - 1
\]

```matlab
x = 0:M-1;
u = 0:M-1;

% Create an input series f(1), ..., f(M)

% 
for n=1:M
    F(n) = 0;
    for m = 1:M
        F(n) = F(n) + f(m) * exp(-j*2*pi*u(n)*x(m)/M);
    end
end
```
Example 5 (continued)

• Show that applying the inverse DFT to F yields the original series f

\[ f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \quad \text{for } x = 0,1,\ldots, M - 1 \]

```matlab
for n=1:M
    f2(n) = 0;
    for m = 1:M
        f2(n) = f2(n) + F(m) * exp(j*2*pi*u(n)*x(m)/M);
    end
end
```
Example 5 (continued)

- Repeat example, using Matlab’s `fft`, `ifft` functions

```matlab
clear all
close all

M = 10;
x = 0:M-1;
u = 0:M-1;

% Create an input series f(1), … ,f(M)
f = rand(1,M)

for n=1:M
    F(n) = 0;
    for m = 1:M
        F(n) = F(n) + f(m) * exp(-j*2*pi*u(n)*x(m)/M);
    end
end
disp(F)

for n=1:M
    f2(n) = 0;
    for m = 1:M
        f2(n) = f2(n) + F(m) * exp(j*2*pi*u(n)*x(m)/M);
    end
end
disp(f2)

F2 = fft(f);
disp(F2)
f3 = ifft(F2);
disp(f3)
```
Example 6

- Fourier says that any function $f(x)$ can be represented as a sum of waves
  
  $$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi u x / M} \quad \text{for } x = 0,1,\ldots, M - 1$$

- Each wave is $e^{j2\pi x / M}$. What do these waves look like?

```matlab
M = 100;
x = 0:M-1;
u = 0:M-1;

for m=1:M
    f = exp(j*2*pi*u(m)*x/M);
    subplot(1,2,1), plot(x,real(f)), title('Real');
    subplot(1,2,2), plot(x,imag(f)), title('Imag');
    freq = u(m)/M;
wavelength = 1/freq;
    fprintf('u = %d, freq = %f, wavelength = %f
', u(m), freq, wavelength);
    pause
end
```
Example 7

• Show that the 1D discrete Fourier transform is a linear operation.

• Linearity:

\[ af_1(x) + bf_2(x) \leftrightarrow aF_1(u) + bF_2(u) \]