Deconvolution

Examples
Example 1

• An image is degraded by a Gaussian blur whose Fourier transform $H(u,v)$ is a Gaussian

$$H(u) = e^{-\left(\frac{u^2+v^2}{2\sigma^2}\right)}$$

• where $\sigma = 20$. A one-dimensional cross section of $H$ through the $u$-axis is shown below.
Example 1 (continued)

• The blurred image is corrupted by noise, with power spectrum $S_\eta (u,v)$. Assume that spectrum $S_\eta (u,v) >> S_f (u,v)$ for values of $(u^2 + v^2) > 2\sigma^2$ where $S_f$ is the power spectrum of the original image.

• Also, $S_\eta (u,v) << S_f (u,v)$ for all values of $(u^2 + v^2) \leq 2\sigma^2$, except for the frequencies $(u,v) = (\pm \sigma, 0)$. At those frequencies, the noise is higher due to an additional periodic noise component), and $S_\eta (u,v) \approx S_f (u,v)$.
Example 1 (continued)

- A Wiener filter $R_{W}(u,v)$ is used to restore the signal, such that

$$\hat{F}(u,v) = R_{W}(u,v)G(u,v)$$

- where $\hat{F}(u,v)$ is the estimated Fourier transform of the restored signal and $G(u,v)$ is the Fourier transform of the measured (degraded) signal. Recall that $R_{W}(u,v)$ is defined as

$$R_{W}(u,v) = \left[ \frac{H^*(u,v)}{[H(u,v)]^2 + S_{\eta}(u,v)/S_{f}(u,v)} \right]$$

- What are the values of $R_{W}(u,v)$ at $(u,v) = (0,0)$ and $(u,v) = (\sigma,0)$?

- Sketch a cross section of $R_{W}(u,v)$ through the $u$-axis.
Example 2

• An image is blurred by atmospheric turbulence, according to the model given in equation 5.6.3 in the book.
  – Give the expression for a Weiner filter, assuming that the ratio of the power spectra of the noise and undegraded signal is a constant.
  – Plot the spectrum of the filter.

\[ H(u, v) = e^{-k(u^2 + v^2)^{5/6}} \]

• An observation:
  – The deblurring filter should enhance higher frequencies, thus undoing the effect of the blurring filter, which depresses the higher frequencies.
  – However, at very high frequencies, it should go to zero, because the noise in the original image is much larger than the signal at those frequencies.
Example 2 (continued)

• We have:

\[
W(u, v) = \frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K}
\]

• For atmospheric turbulence,

\[
H(u, v) = e^{-k(u^2 + v^2)^{5/6}}
\]

• So

\[
W(u, v) = \frac{H(u, v)}{H^2(u, v) + K} = \frac{e^{-k(u^2 + v^2)^{5/6}}}{e^{-2k(u^2 + v^2)^{5/6}} + K}
\]
Example 2 (continued)

- Plot the spectrum:

```matlab
clear all
close all

N = 400;    % Size of image is N x N
k = 0.001;  % Mild turbulence, according to the book
K = 0.001;  % Estimate noise to signal ratio

[U,V] = meshgrid(-N/2:N/2, -N/2:N/2);
W = exp(-k*(U.^2 + V.^2).^(5/6)) ./ ( exp(-2*k*(U.^2 + V.^2).^(5/6)) + K );
imshow(W,[]);
```

- You can also plot $W(u,v)$ as a surface, but plotting is slow unless you subsample:

```matlab
[U,V] = meshgrid(-N/2:8:N/2, -N/2:8:N/2);
W = exp(-k*(U.^2 + V.^2).^(5/6)) ./ ( exp(-2*k*(U.^2 + V.^2).^(5/6)) + K );
figure, meshc(U,V,W);
```
Example 3

• The image “blurred.tif” on the course website has been blurred by a Gaussian blur of unknown size. Using the method of Weiner filtering, deblur this image as best as you can.

```matlab
clear all
close all

% Read in blurred image; convert to type double; range is 0..1
g = im2double(imread('blurred.tif'));
figure(2), imshow(g, []), impixelinfo;
```
Example 3 (continued)

- As in the lecture, let’s choose \( K = 1e^{-4} \).
- We’ll try different values of sigma for the Gaussian and see which one deblurs the best.

```matlab
K = 1e-4;

% Ok, try constructing the ideal gaussian blur filter
for sigma=0.5:0.1:2.5
    h = fspecial('gaussian', 2*ceil(3*sigma)+1, sigma);

    % Call Weiner filter deconvolution function.
    % Note - if you pass in image of type "double" for g, this function
    % will scale it to 0..1. So make sure your NSR assumes that too.
    fRestore = deconvwnr(g,h,K);

    figure(3), imshow(fRestore,[]), title(sprintf('sigma = %f', sigma)), impixelinfo;
    pause
end
```
Example 4

• The image “car.jpg” on the course website is degraded by a motion blur.

• Try to estimate the approximate length and angle of the blur by inspecting the image, using Matlab’s imtool or imshow

• Restore the image using the Weiner filter (Matlab’s deconvwnr function).
  – Try different values for the length and angle of the blurring operator to obtain the best restored image.
  – What does the license plate say?