Sample Problem Set

The exam covers these sections in the Gonzalez and Woods textbook: Ch 2, 3.1-3.6, 4.1-4.10, 9.1-9.3, 10.1-10.2.6. It will be closed book, but handwritten notes are allowed. The problems below are representative of exam problems (although there may be more problems than would appear on the actual exam). Some of the problems below are drawn from previous exams.

For reference, here are some important properties and equations (this will be provided):

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier transform</td>
<td>[ F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ax/M + vy/N)} ] (discrete)</td>
</tr>
<tr>
<td></td>
<td>[ F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ax + vy)} , dx , dy ] (continuous)</td>
</tr>
<tr>
<td>Inverse Fourier transform</td>
<td>[ f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ax/M + vy/N)} ] (discrete)</td>
</tr>
<tr>
<td></td>
<td>[ f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ax + vy)} , du , dv ] (continuous)</td>
</tr>
<tr>
<td>Important Fourier pairs</td>
<td>(impulse) \quad \delta(x,y) \quad \leftrightarrow \quad 1</td>
</tr>
<tr>
<td></td>
<td>(Gaussian) \quad \frac{1}{2\pi\sigma^2} e^{-(x^2 + y^2)/2\sigma^2} \quad \leftrightarrow \quad e^{-2\pi^2\sigma^2(u^2 + v^2)}</td>
</tr>
<tr>
<td></td>
<td>(rect funct) \quad \text{rect}[a,b] \quad \leftrightarrow \quad ab \frac{\sin(\pi u a)}{(\pi u a)} \frac{\sin(\pi v b)}{(\pi v b)} e^{-j\pi (ua + vb)}</td>
</tr>
<tr>
<td>Convolution</td>
<td>[ f(x,y) \ast h(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) h(x - m, y - n) ]</td>
</tr>
<tr>
<td>Correlation</td>
<td>[ f(x,y) \otimes h(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f^*(m,n) h(x + m, y + n) ]</td>
</tr>
</tbody>
</table>
| Translation                  | \[ f(x - x_0, y - y_0) \quad \leftrightarrow \quad F(u,v) e^{-j2\pi(x_0u/M + y_0v/N)} \]
|                              | \[ F(u - u_0, v - v_0) \quad \leftrightarrow \quad f(x,y) e^{j2\pi(u_0x/M + v_0y/N)} \] |
| Scaling                      | \[ a f(x,y) \quad \leftrightarrow \quad a F(u,v) \]
|                              | \[ f(ax,by) \quad \leftrightarrow \quad \frac{1}{|ab|} F(u/a,v/b) \] |
| Rotation                     | \[ x = r \cos \theta \quad y = r \sin \theta \quad u = r \cos \phi \quad w = r \cos \phi \]
|                              | \[ f(r,\theta + \theta_0) \quad \leftrightarrow \quad F(\omega, \phi + \theta_0) \] |
| Morphological erosion        | \[ A \quad \Theta \quad B = \{ z \mid (B)_z \subseteq A \} \quad \text{or} \quad A \quad \Theta \quad B = \{ z \mid (B)_z \cap A^c \neq \emptyset \} \] |
| Morphological dilation       | \[ A \quad \Theta \quad B = \{ z \mid (\tilde{B})_z \cap A \neq \emptyset \} \] |
1. The image of stripes below on the left is too dark when displayed on a monitor. To improve the display, we apply a gamma transformation to produce the image on the right.

![Image of stripes]  
![Image of enhanced stripes]

Sketch the gray level transformation for a gamma transformation that will produce the image on the right.

2. A digital camera captures a 400x400 pixel image of a black and white soccer ball on a gray background. The ball is 30 cm in diameter and is 3 m away from the camera. The camera has a field of field of 90 degrees horizontally and vertically.

(a) What is the diameter of the ball in pixels, in the image?
   (i) 3 pixels (ii) 10 pixels (iii) 20 pixels (iv) 31 pixels (v) 63 pixels

(b) Give the histogram of the image, assuming that black, white, and gray pixels have the gray levels 0, 255, and 128, respectively. Assume that there are an equal number of black and white pixels on the ball.

(c) A ping-pong (table tennis) ball is 40 mm in diameter. How far away must the ping-pong ball be, to be exactly the same apparent size as the soccer ball in the image?
   (i) 10 cm (ii) 0.4 m (iii) 0.75 m (iv) 1 m (v) 3 m

3. A CCD camera has a image resolution of 2000 x 2000 pixels. The individual sensor elements are squares measuring 10 x 10 um, with no spaces between them.
   a. If the camera uses a lens with focal length = 100 mm, what is the field of view?
   b. Assume the camera is mounted on an airplane, pointing straight down at the ground. What height should the airplane fly, so that one pixel in the camera corresponds to one meter on the ground?
4. The histogram of a 100x100 image with 3 bit pixels is given below.

\[ H(r) = \begin{cases} 
2000 & 0 \leq r \leq 3 \\
500 & 4 \leq r \leq 7 
\end{cases} \]

a. Compute the transformation function \( s = T(r) \) that will equalize the histogram.
b. Compute the histogram \( H(s) \) of the resulting image, if it were transformed by \( T \).

5. Complete the following Matlab code to do a correlation between a 10x10 template image \( T \) and a 100x100 image \( I \) (you do not have to normalize the result).

```matlab
% Assume that I, T are already loaded
corr = zeros(100); % resulting correlation image
for r=1:90
    for c=1:90
        % Compute the score for this point (r,c)
        : (your code goes here)
        corr(r,c) = score;
    end
end
```

6. Compute the convolution of the following image with the filter \( w \) (assume that the origin of \( w \) is at its center). To compute the result at the border, pad the image with zeros.

\[
w = \begin{bmatrix}
-1 & -1 & -1 \\
0 & 0 & 0 \\
1 & 1 & 1
\end{bmatrix}
\]
7. Consider a 3x3 spatial mask that averages the four closest neighbors of a point \((x,y)\) but excludes the point itself from the average. Find the equivalent filter, \(H(u,v)\) in the frequency domain. (Hint: Use the translation property of the Fourier transform).

8. You want to apply the Laplacian of a Gaussian edge operator, \(\nabla^2 G\) to an image \(f(x, y)\) of size 512x512. Assume that the edge operator is of size 32x32, and that its origin is at its center. Describe in words how to perform this in the frequency domain.
9. Describe the Fourier transform of a filter that performs smoothing with the ideal low pass filter, followed by differentiation in the x direction. Sketch a plot of its magnitude.

Notes: The Fourier transform of the ideal low pass filter is:

\[
H(u,v) = \begin{cases} 
1 & u^2 + v^2 \leq R^2 \\
0 & u^2 + v^2 > R^2 
\end{cases}
\]

The Fourier transform equivalent of differentiation is

\[
\frac{\partial^n f(x,y)}{\partial x^n} \iff (j2\pi u)^n F(u,v)
\]

Smoothing followed by differentiation is

\[
\left(\frac{\partial}{\partial x}\right)^n (h * f) = \left(\frac{\partial}{\partial x} * h\right) * f
\]

10. Assume that the 2D Fourier transform of the MxN image \( f(x,y) \) is \( F(u,v) \). Determine the 2D Fourier transform of the following functions. You may wish to refer to the properties of the Fourier transform in the table on the front page.

a. \( 4f(2x - 4, y + 2) \)

   (select one)
   (i) \( 2e^{-j2\pi(2u/M-2v/N)} F(u/2,v) \)
   (ii) \( 4e^{-j2\pi(2u/M-2v/N)} F(u/2,v) \)
   (iii) \( 2e^{-j2\pi(4u/M-2v/N)} F(u/2,v/2) \)
   (iv) \( 4e^{-j2\pi(u/M-2v/N)} F(u,v/2) \)

b. \( f(x - 3, y + 2) * f(x + 3, y) \) (where * denotes convolution)

   (select one)
   (i) \( e^{j2\pi(v/N)} F(u,v) \)
   (ii) \( F^2(u,v) \)
   (iii) \( e^{j2\pi(u/M-2v/N)} F^2(u,v) \)
   (iv) \( e^{j2\pi(2v/N)} F^2(u,v) \)
11. The following is a binary image $I$ and a structuring element $S$. Assume that the origin of $S$ lies at its center.

(a) Predict the result of dilating $I$ with $S$.

(b) Predict the result of eroding $I$ with $S$.

12. A small portion of a binary image $I$ is shown below. Assume that you want to join the two groups of 1’s into a single connected group, while distorting the overall image as little as possible (this might be from an image of a fingerprint, for example).

a. Using the structuring element $S$ shown below, would you use morphological opening or morphological closing?

b. Apply the operation you selected in (a) to the image $I$. (Assume that there are 0’s outside the border of the image.) Show the intermediate results of dilation and erosion (or vice versa).
13. A camera takes images of a test pattern. The image is converted to binary, as shown in the left image. Occasionally, diagonal streaks appear in the image due to some unknown noise process, as shown in the right image. To help diagnose the problem, we want to create an algorithm that will automatically count the streaks. It is observed that the streaks are always at the same angle (45 degrees), between 35 and 50 pixels long, and between 2 and 4 pixels wide. Describe an algorithm, based on morphological processing, that will automatically count the streaks. (Be as specific as possible!)

14. Explain why the edges found by the Laplacian of a Gaussian edge operator form closed contours.