P3P
Pose from 3 Point Correspondences

• Pose can be computed from 3 points using a closed-form (non-iterative) algorithm
  – Is called the perspective-3-point-problem (P3P)

• Basic observation:
  – The visual angle between any pair of 2D points must be the same as the angle between their corresponding 3D points

\[
\begin{align*}
\theta_{ij} & = \cos^{-1}\left( \frac{d_{ij}^2 - d_i^2 - d_j^2}{2d_i d_j} \right) \\
\end{align*}
\]

The law of cosines:

\[
d_{ij}^2 = d_i^2 + d_j^2 - 2d_i d_j \cos \theta_{ij}
\]

Knowns: \(d_{ij}, \theta_{ij}\)

Unknowns: \(d_i, d_j\)

We assume that points are not colinear.

\[ s_1^2 = L_A^2 + L_B^2 - L_A L_B \cos \theta_{AB} \]
\[ s_2^2 = L_B^2 + L_C^2 - L_B L_C \cos \theta_{BC} \]
\[ s_3^2 = L_A^2 + L_C^2 - L_A L_C \cos \theta_{AC} \]

- Fischler and Bolles show how the lengths \( L_A, L_B, L_C \) can be found by solving for the roots of a 4\(^{th} \) order polynomial.

\[ G_0 + G_1 x + G_2 x^2 + G_3 x^3 + G_4 x^4 = 0 \]

Fischler and Bolles, “Random Sample Consensus”
Communications of the ACM, Vol. 24, Number 6, June, 1981
• With 3 points, we can have up to four solutions

Can use more (n>3) points to eliminate ambiguity (6 points will have no ambiguity)

• Still need to solve for pose (rotation, translation), given the two sets of corresponding 3D points
• The problem of solving for the 3D transformation that aligns two sets of corresponding 3D points is called the “absolute orientation” problem
• It is relatively straightforward to solve in closed form