 Recovery of Periodic Clustered Sparse Signals From Compressive Measurements

Chia Wei Lim and Michael B. Wakin
Department of Electrical Engineering and Computer Science
Colorado School of Mines

Abstract—The theory of Compressive Sensing (CS) enables the efficient acquisition of signals which are sparse or compressible in an appropriate domain. In the sub-field of CS known as model-based CS, prior knowledge of the signal sparsity profile is used to improve compression and sparse signal recovery rates. In this paper, we show that by exploiting the periodic support of Periodic Clustered Sparse (PCS) signals, model-based CS improves upon classical CS. We quantify this improvement in terms of simulations performed with a proposed greedy algorithm for PCS signal recovery and provide sampling bounds for the recovery of PCS signals from compressive measurements.

Index Terms—Compressive Sensing, model-based Compressive Sensing, Periodic Clustered Sparse signals

I. INTRODUCTION

Compressive Sensing (CS) has recently enabled efficient signal acquisition protocols with acquisition requirements much lower than those required by the Nyquist sampling theorem [1, 2]. CS exploits the compressibility of signals that can be well represented by just a few coefficients in an appropriate basis. In the classical setting, CS acquisition protocols involve taking a small number of “compressive” linear measurements of a signal and subsequently recovering the signal from such measurements by solving a convex optimization problem or using a greedy algorithm.

More recently, it has been shown [3] that by imposing further structural dependencies in the locations of the signal coefficients, the number of compressive measurements required for signal recovery can be further reduced. These structural dependencies require a priori information about the structure of the signal support, and therefore signal recovery that relies on such information is termed model-based CS.

In this paper, the theory of model-based CS [3] is used to reduce the number of compressive measurements required for recovery of Periodic Clustered Sparse (PCS) signals. PCS signals, which we define formally in Section III, are sparse signals whose supports are concentrated in a periodically repeating cluster. The model-based CoSaMP [3] algorithm, incorporated with a new model-based approximation for the PCS signal model, is shown to outperform CoSaMP [4]. The proposed PCS model-based approximation algorithm estimates key parameters associated with the PCS signal on the fly.

PCS signals can often be found in Time Division Multiple Access (TDMA) networks having low traffic density. Typically TDMA networks allow multiple users to transmit on the same frequency channel by allocating specific time slots to its users. During periods of low traffic intensity, where only one of many users are transmitting, the PCS signal aptly models such TDMA networks.

The rest of this paper is organized as follows. Section II briefly reviews the theory of CS and model-based CS. Section III introduces the PCS signal model. In Section IV, model-based CoSaMP is first introduced and is then followed by a discussion of a proposed PCS model-based approximation algorithm which is directly incorporated into the model-based CoSaMP algorithm for signal recovery. Section IV continues with the derivation of sampling bounds using subspace counting arguments. Finally, simulation results based on the PCS adapted model-based CoSaMP algorithm and concluding remarks are found in Sections V and VI, respectively.

II. PRELIMINARIES

A. CS Preliminaries

Sparsity and compressibility are key notions fundamental to CS signal acquisition. These concepts can be explained as follows. Given a signal vector $x \in \mathbb{R}^N$ and an orthonormal basis $\Psi \in \mathbb{R}^{N \times N}$, the signal’s coefficients (representation) $\eta \in \mathbb{R}^N$ in $\Psi$ are related via $x = \Psi \eta$. A signal $x$ is said to be $s$ sparse in $\Psi$ if $\eta$ contains only $s \ll N$ nonzero entries and compressible in $\Psi$ if $\eta$ is dominated by a few large entries.

Most CS acquisition protocols obtain compressive linear measurements $y \in \mathbb{R}^M$ of $x$ via $y = \Phi x = \Phi \Psi \eta = A \eta$ where the sensing matrix $\Phi \in \mathbb{R}^{M \times N}$ is typically constructed randomly, $A = \Phi \Psi$ and $M \ll N$. A matrix $A$ is said to satisfy the Restricted Isometry Property (RIP) of order $s$ with isometry constant $\delta_s \in (0,1)$ if $(1 - \delta_s) \| \eta \|_2^2 \leq \| A \eta \|_2^2 \leq (1 + \delta_s) \| \eta \|_2^2$ holds for all $s$ sparse vectors $\eta$. If $A$ satisfies the RIP, recovery of an $s$ sparse signal $x \in \mathbb{R}^N$ from the compressive measurements $y$ is possible [5]. For any fixed orthonormal basis $\Psi$, if $M = \mathcal{O}(s \log(\frac{N}{s}))$ and the sensing matrix $\Phi$ is generated randomly with independent and identically distributed (IID) Gaussian or Bernoulli ($\pm \frac{1}{\sqrt{M}}$) entries, then $A$ will satisfy the RIP with high probability.

B. Model-based CS Preliminaries

An $s$ sparse coefficient vector $\eta \in \mathbb{R}^N$ lives in a union of $m_s = \binom{N}{s}$ subspaces of dimension $s$. When additional a priori information on the support of $\eta$ is available, the number of possible subspaces $m_s$ can be reduced. This reduction
allows for an improvement upon conventional CS in terms of the number of measurements required for signal recovery. Formally, such a priori information is referred to as a signal model $\mathcal{M}_s$ and is defined [3] as

$$\mathcal{M}_s = \bigcup_{m=1}^{m_s} \mathcal{X}_m, \mathcal{X}_m := \{ \eta : \eta|_{\Omega_m} \in \mathbb{R}^s, \eta|_{\Omega^C_m} = 0 \},$$

(1)

where $\eta|_\Omega$ are the entries of $\eta$ with indices $\Omega \subseteq \{1, \ldots, N\}$, $\Omega^C$ is the complement of $\Omega$, and each subspace $\mathcal{X}_m$ in (1) contains all $\eta$ supported on $\Omega_m$. In other words, the index sets $\Omega_m$ refer to all supports allowed in $\mathcal{M}_s$. All coefficient vectors $\eta$ belonging to such a model $\mathcal{M}_s$ (including the PCS signals to be discussed in Section III) are referred to as s-model sparse [3].

For a collection of s-model sparse signals, a matrix $A$ is said to obey the model-based RIP [6, 7] if

$$(1 - \delta_{\mathcal{M}_s})||\eta||_2^2 \leq ||A\eta||_2^2 \leq (1 + \delta_{\mathcal{M}_s})||\eta||_2^2$$

(2)

holds for all $\eta \in \mathcal{M}_s$ with model-based isometry constant $\delta_{\mathcal{M}_s}$. Blumensath et al. [6] proved an $M \times N$ i.i.d. subgaussian random matrix $A$ satisfies the model-based RIP with probability at least $1 - e^{-t}$ if

$$M \geq \frac{2}{C\delta_{\mathcal{M}_s}^2} \left[ \ln(2m_s) + \ln \frac{12}{\delta_{\mathcal{M}_s}} + t \right].$$

(3)

In conventional CS where $m_s = \binom{N}{s}$, substituting this value of $m_s$ into (3) gives $M = \mathcal{O}(s \log \binom{N}{s})$. Eqn. (3) shall be used to derive the sampling bound required for successful PCS signal recovery from compressive measurements in Section IV.

III. PCS SIGNAL MODEL

In this paper, a PCS signal $\eta \in \mathbb{R}^N$ is one that is s sparse, parameterized by a fixed period $L \in \{1, 2, \ldots, N\}$, start-phase $\alpha \in \{1, 2, \ldots, L\}$, and cluster size $D \in \{1, 2, \ldots, L\}$. All indices $\beta$ belonging to the support of such a PCS signal obey $\beta = \alpha + (i - 1) + jL$, where all values of $i \in \{1, 2, \ldots, D\}$ and $j \in \mathbb{Z}$ are taken such that $1 \leq \beta \leq N$. In other words, a PCS signal is simply one whose support has a periodic cluster structure with start-phase $\alpha$, period $L$, and duty-cycle $\frac{D}{L} \approx \frac{1}{N}$. A typical PCS signal sparsity profile is shown in Fig. 2(a). Evidently, due to the periodic structure of the sparsity profile of the PCS signal, such a priori information can be further used to improve the recovery of PCS signals from compressive measurements. This will be further discussed in the next section. The PCS signal can be seen as a special case of the $(s, C)$-clustered sparse signal introduced in [8]. However, as we discuss in Section IV-C, and in contrast to what one would obtain by applying the results in [8], the sampling bound we derive for PCS signals is independent of the number of clusters $C$.

In this paper we will assume that $s$ and $N$ are known for a given signal, but that $L$, $\alpha$, and $D$ are unknown. In some cases, however, we may have prior information that the period $L \in \{L_{\min}, \ldots, L_{\max}\}$, for some minimum and maximum possible periods $L_{\min}$ and $L_{\max}$, respectively. In general, if $L_{\min}$ and $L_{\max}$ are unknown, appropriate values would be $L_{\min} = 1$ and $L_{\max} = \frac{N}{2}$ (since a minimum of two consecutive periods is required to determine the period). We note that although $L$, $\alpha$, and $D$ may all be unknown a priori, since $s$ and $N$ are known and $\frac{D}{L} \approx \frac{1}{N}$, only $\alpha$ and $L$ remain as free parameters defining a PCS signal. As such the PCS signal model $\mathcal{M}^\text{PCS}_s$ can be formulated as

$$\mathcal{M}^\text{PCS}_s = \bigcup_{m=1}^{m_{\text{PCS}}} \mathcal{X}(\alpha_m, L_m) = \bigcup_{L=L_{\min}}^{L_{\max}} \left( \bigcup_{\alpha=1}^{L} \mathcal{X}(\alpha, L) \right),$$

(4)

where $m_{\text{PCS}}$ is the number of possible subspaces of the PCS signal and $\mathcal{X}(\alpha, L)$ is a subspace corresponding to the support having start-phase $\alpha$ and period $L$.

IV. PCS SIGNAL RECOVERY FROM COMPRESSIVE MEASUREMENTS

A. Model-Based CoSaMP

In the CS signal recovery problem, one wishes to recover the entries of $\eta \in \mathbb{R}^N$ given the measurements $y$ and the matrix $A$. Given further the restricted union of subspaces $\mathcal{M}^\text{PCS}_s$, one could solve the following optimization problem:

$$\eta = \arg \min ||\hat{\eta}||_0 \quad \text{s.t.} \quad y = A\hat{\eta} \quad \text{and} \quad \hat{\eta} \in \mathcal{M}^\text{PCS}_s.$$  

(5)

Solving (5) can be achieved by solving a least squares problem for every candidate support and returning the $\hat{\eta}$ corresponding to the minimum least squares error. For the PCS model, we note that solving (5) in this manner has a worst case complexity of $\mathcal{O}(N^2)$ iterations (with each iteration involving a least squares problem), which may be prohibitive when $N$ is large. Therefore faster algorithms are sought in the sequel.

The model-based CoSaMP [3] algorithm, which is based on the CoSaMP [4] algorithm, belongs to the class of greedy algorithms frequently used for CS signal recovery. Due to its established robust signal recovery guarantees and simple iterative greedy structure, model-based CoSaMP shall be used as the primary CS recovery algorithm in this paper. Its pseudo-code is listed in Algorithm 1.

**Algorithm 1 Model-based CoSaMP**

**Inputs:** CS matrix $A$, measurements $y$, model $\mathcal{M}_s$

**Output:** s-sparse approximation $\hat{\eta}$ to true coefficients $\eta$

1: $\hat{\eta}_0 = 0, r = y, T = 0, i = 0 \{\text{initialize}\}$

2: while halting criterion false do

3: $i \leftarrow i + 1$

4: $e \leftarrow A^Tr \{\text{form coefficients residual estimate}\}$

5: $\Omega \leftarrow \supp(M^2(e, s)) \{\text{estimate support}\}$

6: $T \leftarrow \Omega \cup \supp(\hat{\eta}_{i-1}) \{\text{merge supports}\}$

7: $b|_T \leftarrow A^Ty, b|_{\text{RC}} \leftarrow 0 \{\text{form coefficients estimate}\}$

8: $\hat{\eta}_i \leftarrow M_1(b, s) \{\text{prune coefficients estimate}\}$

9: $r \leftarrow y - A\hat{\eta}_i \{\text{update residual}\}$

10: end while

11: return $\hat{\eta} \leftarrow \hat{\eta}_i$

Model-based CoSaMP, being a modified CoSaMP algorithm, distinguishes itself through the signal support estimation...
steps 5 and 8 (in Algorithm 1) which are based on \( M_s \). For a positive integer \( B \), \( M_B(\eta, s) \) is defined [3] as
\[
M_B(\eta, s) = \arg \min_{\eta \in M_s^B} ||\eta - \hat{\eta}||_2,
\]
where \( M_s^B \) is the \( B \)-Minkowski sum [3] for the set \( M_s \):
\[
M_s^B = \left\{ \eta = \sum_{r=1}^{B} \eta^{(r)}, \text{ with } \eta^{(r)} \in M_s \right\}.
\]

Note that when \( B = 1 \), \( M_s^B = M_s \). In words, \( M_B(\eta, s) \) is the model-based approximation algorithm which returns the best approximation of \( \eta \) in the enlarged union of subspaces \( M_s^B \). In the next section, the design of an \( M^B_{\text{PCS}}(\eta, s) \) (based on \( M^B_{\text{PCS}} \)) is discussed. Due to its high complexity, an alternative heuristic approximation \( \hat{M}^B_{\text{PCS}}(\eta, s) \) is proposed with lower complexity.

B. PCS Model-based Approximation Algorithm

The PCS signal model \( M_{\text{PCS}}(\eta, s) \) can be used to gain further intuition as to how a PCS model-based approximation algorithm \( M^B_{\text{PCS}}(\eta, s) \) should be designed. Intuitively, with \( B = 1 \), \( M_{\text{PCS}}(\eta, s) \) should search over a range of \( \alpha \)'s and \( L \)'s and return the approximate support set defined by an \( \alpha_{\text{opt}} \) and an \( L_{\text{opt}} \) which captures most of the energy of \( \eta \). Thanks to the periodic structure of the PCS signal support, such a search can be implemented with a worst case complexity of \( O((L_{\text{max}} - L_{\text{min}} + 1)^2) \). If \( L_{\text{min}} \) and \( L_{\text{max}} \) are unknown, the complexity increases to \( O(N^2) \).

When \( B = 2 \), \( M^2_{\text{PCS}}(\eta, s) \) should return the best approximation to \( \eta \) based on candidate supports generated by union of two PCS signals. Hence, a natural extension of \( M^2_{\text{PCS}}(\eta, s) \) to \( M^B_{\text{PCS}}(\eta, s) \) would be to jointly search over a range of \( \alpha \)'s and \( L \)'s (for PCS signal 1) and over a second range of \( \alpha \)'s and \( L \)'s (for PCS signal 2), and return the support (generated by the union of these individual supports) which captures most of the energy of \( \eta \). Such a search has a worst case complexity of \( O((L_{\text{max}} - L_{\text{min}} + 1)^3) \), or \( O(N^4) \) if \( L_{\text{min}} \) and \( L_{\text{max}} \) are unknown. In general, these search-based \( M^B_{\text{PCS}}(\eta, s) \) algorithms have a worst case complexity of \( O(N^{2B}) \).

Algorithm 2 lists the pseudo-code of a heuristic approximation algorithm \( \hat{M}^B_{\text{PCS}}(\eta, s) \) that has lower computational complexity compared to the optimal \( M^B_{\text{PCS}}(\eta, s) \) previously discussed. Complexity reduction is achieved by observing that due to periodic support of the PCS signal, the support of its signal proxy \( A^T y \) is also periodic. Subsequent Fourier analysis on the energy of \( A^T y \) results in few significant peaks occurring at frequencies \( f = \frac{\ell}{T}, \ j \in \mathbb{Z} \). Therefore, FFT-based methods can be used to decouple the PCS period estimation and PCS start-phase estimation (cf. \( M^B_{\text{PCS}}(\eta, s) \)). Using a variant of the harmonic comb [9], the PCS signal period \( \hat{L}_B \) can be estimated using Algorithm 2 line 6 where \( h_{\text{num}} \) is the number of harmonics used for maximization, and \( h_{\text{min}} \) and \( h_{\text{max}} \) is the minimum and maximum harmonics considered, respectively. If a priori information of the range of \( L \)'s is available, the accuracy of \( \hat{L}_B \) can be improved by restricting \( h_{\text{min}} \) and \( h_{\text{max}} \).

**Algorithm 2 PCS Model-based Approximation \( \hat{M}^B_{\text{PCS}}(\eta, s) \)**

**Inputs:** nominal duty-cycle \( \frac{\ell}{2T} \), input signal \( \eta \), parameter \( B \), FFT length \( N_{\text{fft}} \), range of harmonics \( h_{\text{min}}, h_{\text{max}} \), number of harmonics \( h_{\text{num}} \)

**Output:** approximation to \( \text{supp}(\eta) \)

1. \( \beta = 0 \) \{initialize\}
2. for \( b = 1 \) to \( B \) do
3. \( \eta_{bc} \leftarrow \eta - 0 \) \{compute ony on reduced support\}
4. \( \eta_{bc} \leftarrow abs(\eta_{bc})^2 \) \{compute signal energy\}
5. \( H[k] = abs \left( \sum_{n=0}^{\frac{N_{\text{fft}}}{2}} \eta_{bc}[n]e^{-j2\pi k n} \right) \) \{compute FFT\}
6. \( \hat{k}_{0,b} \leftarrow \arg \max_{k_{0,b} \in [h_{\min}, h_{\max}]} \{ \sum_{a=0}^{h_{\text{num}}} H[a k_{0,b}] \} \) \{estimate PCS frequency\}
7. \( \hat{L}_b \leftarrow \frac{N_{\text{fft}}}{\hat{k}_{0,b}} \) \{estimate PCS period\}
8. \( \hat{D}_b \leftarrow \text{round} \left( \frac{\hat{L}_b}{2} \right) \) \{compute block length\}
9. \( \eta_{ac}[k] \leftarrow \frac{1}{2} \sum_{j=-\hat{D}_b}^{\hat{D}_b} \eta_{bc}(a \hat{L}_b + k) \) \{compute energy average\}
10. \( \hat{a}_b \leftarrow \arg \max_k \eta_{ac} \ast g_{\text{MA}} \) \{estimate PCS start-phase\}
11. \( \beta_b \leftarrow \hat{a}_b + (\ell - 1) + j \hat{L}_b \) \{generate PCS support based on start-phase \( \hat{a}_b \) and period \( \hat{L}_b \)\}
12. \( \beta \leftarrow \beta \cup \beta_b \) \{union PCS support\}
13. end for
14. return \( \beta \) \{estimated support\}

Then, using the estimated period \( \hat{L}_B \), PCS start-phase estimation can be achieved via a series of operations (Algorithm 2 lines 8–10) which obtain the start-phase corresponding to the maximum average signal energy \( \eta_{ac} \) captured assuming a PCS signal template with period \( \hat{L}_B \) and duty-cycle \( \hat{D}_B \). \( \eta_{ac} \) is easily computed by averaging \( \eta_{ac} \) across segments of length \( \hat{L}_B \) (line 9). Next, \( \eta_{ac} \) is filtered using a moving-average (MA) filter \( g_{\text{MA}} \) of length \( \hat{D}_B \). The PCS start-phase \( \hat{a}_b \) is estimated as the time index corresponding to the maximum value of the output of the MA filter. Overall, the algorithm loops over \( B \) total estimates of PCS signals, and in line 1 the supports of the previous estimates are zeroed out.

The complexity of Algorithm 2 can be derived as follows: the complexity of the FFT is \( O(N \log N) \), the worst case complexity of the harmonic comb is \( O(N) \), the worst case complexity of the averaging operation is \( O(N) \) and the worst case complexity of moving average filtering is \( O(N \log N) \). Hence the overall worst case complexity of Algorithm 2 is \( O(BN \log N) \). Figure 1 shows a comparison of the timing

![Fig. 1. Timing profile comparison across different algorithms.](image-url)
profile of model-based CoSaMP with $\hat{M}^\text{PCS}_{s,s} (\eta, \delta)$, model-based CoSaMP with $\hat{M}^\text{PCS}_{s,s} (\eta, \delta)$, and $\ell_0$ minimization.

C. Sampling Bound

Given a family of $s$-model sparse signals $\mathcal{M}_s$, model-based CoSaMP can be shown to perfectly recover any $\eta \in \mathcal{M}_s$ from the measurements $y = A\eta$ if the model-based RIP (2) holds for all $\eta \in \mathcal{M}_s$ with constant $\delta_{M^s} \leq 0.1$ [3]. The recovery is also robust with respect to noise. In the context of PCS signals, when $\mathcal{M}_s = M^\text{PCS}$, we can compute the number of measurements $M$ required for an $M \times N$ IID subgaussian matrix $A$ to satisfy the model-based RIP for all $\eta \in \mathcal{M}_s^4$.

Theorem 4.1: With high probability, the number of measurements sufficient for successful recovery of $s$-sparse PCS signals is given by $M^\text{PCS} = \mathcal{O} (s + \log N)$.

Proof: For the PCS model, $m_s$, the number of subspaces of $\mathcal{M}_s$, can be easily derived using the following argument. For a given hypothesized period $L$, the number of possible start-phases is exactly $L$. Therefore $m_s = 1 + 2 + \cdots + \frac{N_s}{2} = \frac{N_s}{2} + 1$, where the following assumptions are made: $L_{\text{min}} = 1$, $L_{\text{max}} = \frac{N_s}{2}$. To compute a bound on $M^\text{PCS}$, we employ (3). However, because we require the model-based RIP for all $\eta \in \mathcal{M}_s^4$ rather than merely all $\eta \in \mathcal{M}_s$, we must substitute $(m_s)^4$ in place of $m_s$ and $4s$ in place of $s$. The final result changes only by a constant.

We note that Theorem 4.1 applies to model-based CoSaMP (Algorithm 1) using $M^\text{PCS} (\eta, s)$ and is not technically guaranteed to apply to model-based CoSaMP using $M^\text{PCS}_{s,c} (\eta, s)$. We also note that the $(s, C)$-clustered sparse signal model $\mathcal{M}_{s,c} (\eta, \delta)$ [8] allows supports which occur in clusters ($C$ is the maximum number of clusters). In contrast to the $(s, C)$-clustered sparse signal sampling bound $M_{s,c} = \mathcal{O} \left( s + C \log \frac{N}{2} \right) [8]$, $M^\text{PCS}$ is not dependent on the number of clusters, thanks to the assumed periodicity of the PCS signal clusters.

V. Simulation Results

In this section, the performance of model-based CoSaMP (Algorithm 1) incorporated with $\hat{M}^\text{PCS}_{s,s}$ (Algorithm 2 executed with $N = 256$, $h_{\text{min}} = 3$, $h_{\text{min}} = 2$ and $h_{\text{max}} = 50$) is compared with that of a model-based CoSaMP incorporated with $M^\text{PCS}$ (executed with $l_{\text{min}} = 30$ and $l_{\text{max}} = 34$), the standard CoSaMP [4], and $\ell_0$ minimization. PCS coefficient vectors $\eta$ with $s = 40$, $L = 32$, $N = 256$ and $\alpha = 10$ were constructed with entries generated from an IID normal distribution; Figure 2(a) shows a typical realization. 500 independent trials were performed using independent realizations of $\eta$ and of the matrix $A$, which contained entries generated from IID Bernoulli ($\pm 1$) distribution. In these simulations, we take $\Psi = I_{N \times N}$, and hence $\eta = x$.

Figure 2(b) shows the probability of successful PCS signal recovery from compressive measurements as a function of $\frac{M}{\eta}$. A recovered signal $\hat{x}$ is considered successfully recovered when $||\hat{x} - x||_\infty \leq 10^{-4}$. We note that as compared to conventional CoSaMP, model-based CoSaMP performs significantly better in terms of PCS signal success recovery rate especially when $\frac{M}{\eta} < 3$. Model-based CoSaMP achieves better than 80% success rate even when $\frac{M}{\eta} = 2$ as compared to CoSaMP which is unable to successfully recover the signal at all. We note that as compared to model-based CoSaMP (with $M^\text{PCS}$), model-based CoSaMP (with $M^\text{PCS}$) offers comparable performance down to $\frac{M}{\eta} = 2.2$, with significant degradation when $\frac{M}{\eta} = 2$. We note that because line 7 of Algorithm 1 involves solving a least squares problem with up to $2s$ unknowns, we did not pursue simulations with $M < 2s$.

Figure 2(c) shows the Normalized Mean Squared Error (NMSE) of the recovered signal $\hat{x}$ as a function of $\frac{M}{\eta}$ where NMSE is computed as $NMSE = \frac{1}{500} \sum_{i=1}^{500} ||\hat{x}_i - x_i||_2^2$. Model-based CoSaMP again performs significantly better than CoSaMP in terms of NMSE especially when $\frac{M}{\eta} < 3$. Model-based CoSaMP (with $M^\text{PCS}$) performs significantly better than model-based CoSaMP (with $M^\text{PCS}$) when $\frac{M}{\eta} < 3$.

VI. Conclusion

Relying on a priori information of the support of the PCS signal, a PCS signal model $\mathcal{M}^\text{PCS}$ (a union of subspaces) was formulated. Then, using insights gained from the $M^\text{PCS}$ formulation, a PCS model-based approximation algorithm $\hat{M}^\text{PCS}$ was designed and a sampling bound for ideal PCS signal recovery was derived. Finally, through Monte Carlo simulations, it was demonstrated that model-based CoSaMP using $\hat{M}^\text{PCS}$ offered comparable performance with model-based CoSaMP using $M^\text{PCS}$ down to $\frac{M}{\eta} = 2$ and still performed better than CoSaMP especially in the regime when $\frac{M}{\eta} < 3$. 

---

1. Here, we denote $M^\text{PCS}_{s,s} (\eta, s)$ by $M^\text{PCS}$ and $M^\text{PCS}_{s,c} (\eta, s)$ by $M^\text{PCS}_{s,c}$.
2. These values were chosen to reduce overall simulation run-time (cf. Fig. 1).
REFERENCES


