MAKING MEASUREMENTS IN QUANTUM MECHANICS

The wavefunction need not be one of the stationary states but can be any function satisfying constraints such as normalizability and continuity. Nonetheless, it can be decomposed into a linear combination (or linear “superposition”) of the stationary states. However, the stationary states, while a special basis because of their role in determining time-dependence, are not the only possible basis. In fact, the eigenfunctions \( \phi_n(r) \) of any operator \( \hat{A} \) corresponding to a measurement of a real quantity (which in fact is equivalent to saying that \( \hat{A} \) is Hermitian) will form an orthonormal basis. Hence any (normalizable, continuous, and otherwise observing the boundary conditions) spatial wavefunction \( \psi(r) \) can be written as the linear superposition

\[
\psi(r) = \sum_n c_n \phi_n(r)
\]

(Note the coefficients \( c_n \) are not the same as the “\( c_n \)” used for an expansion in terms of the stationary states.) Recall that we have

\[
\hat{A}\phi_n(r) = a_n\phi_n(r)
\]

where the \( a_n \) are the only values that can be observed for the operator \( \hat{A} \).

We can then use Postulate 4 to determine the physical meanings of the coefficients \( c_n \):

If a system is in a state described by a normalized wave function \( \Psi \), then the average value of the observable corresponding to \( \hat{A} \) is given by

\[
\langle a \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{A} \Psi \, dr
\]

The wavefunction \( \Psi \) can then be decomposed in terms of the eigenstates of \( \hat{A} \):

\[
\langle a \rangle = \int \sum_m c_m^* \phi_m^*(r) \hat{A} \sum_n c_n \phi_n(r) \, dr
\]

\[
= \sum_{m,n} c_m^* c_n a_n \int \phi_m^*(r) \phi_n(r) \, dr = \sum_{m,n} c_m^* c_n a_n \delta_{m,n}
\]

\[
= \sum_n |c_n|^2 a_n
\]

Thus we see that \( |c_n|^2 \) represent the weight of the possible observable values \( a_n \), where the above sum represents a weighted average over outcomes. Moreover, the \( |c_n|^2 \) represent a valid normalized weight distribution, since it can be shown from the normalization condition on \( \psi(r) \) that

\[
\sum_n |c_n|^2 = 1
\]