Fluid Mechanics FE Review

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MAJOR TOPICS
Fluid Properties
Fluid Statics
Fluid Dynamics
Fluid Measurements
Dimensional Analysis

Fluid Mechanics 9-1a1
Definitions

Fluids
• Substances in either the liquid or gas phase
• Cannot support shear

Density
• Mass per unit volume

Specific Volume
\[ v = \frac{1}{\rho} \]

Specific Weight
\[ \gamma = \lim_{\Delta V \to 0} \left( \frac{g \Delta m}{\Delta V} \right) = \rho g \]

Specific Gravity
\[ SG = \frac{\rho}{\rho_{water}} = \frac{\gamma}{\gamma_{water}} \]
Fluid Mechanics

Definitions

Example (FEIM):
Determine the specific gravity of carbon dioxide gas (molecular weight = 44) at 66°C and 138 kPa compared to STP air.

\[
\text{Ans: } 1.67
\]
Fluid Mechanics 9-1b

Definitions

Shear Stress
- Normal Component: \( \tau_n = p \quad 22.9 \)
- Tangential Component
  - For a Newtonian fluid: \( \tau_t = \mu \frac{dv}{dy} \quad 22.11 \)
  - For a pseudoplastic or dilatant fluid: \( \tau_t = K \left( \frac{dv}{dy} \right)^n \quad 22.12 \)

Fluid Mechanics 9-1c1

Definitions

Absolute Viscosity
- Ratio of shear stress to rate of shear deformation

Surface Tension
\( \sigma = \frac{F}{L} \quad 22.14 \)

Capillary Rise
\( h = \frac{4 \sigma \cos \beta}{\rho g} \quad [\text{SI}] \quad 22.17b \)
Example (FEIM):
Find the height to which ethyl alcohol will rise in a glass capillary tube 0.127 mm in diameter.
Density is 790 kg/m³, \( \sigma = 0.0227 \) N/m, and \( \beta = 0° \).

\[
h = \frac{4\sigma \cos \beta}{\gamma d} = \frac{(4)(0.0227 \text{ kg/s}^2)(1.0)}{790 \text{ kg/m}^3 \left( \frac{9.8 \text{ m}}{\text{s}^2} \right)(0.127 \times 10^{-3} \text{ m})} = 0.00923 \text{ m}
\]

Ans: 0.00923 m
Fluid Statics

Gage and Absolute Pressure

\[ p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atmospheric}} \]

Hydrostatic Pressure

\[ p = \gamma h + \rho gh \]

\[ p_2 - p_1 = -\gamma (z_2 - z_1) \]

Example (FEIM):
In which fluid is 700 kPa first achieved?

\[
\begin{array}{|c|c|c|}
\hline
\text{Depth (m)} & \text{Fluid} & \text{\( \gamma \) (kPa/m)} \\
\hline
60 & \text{ethyl alcohol} & 7.586 \\
10 & \text{oil} & 8.825 \\
5 & \text{water} & 9.604 \\
5 & \text{glycerin} & 12.125 \\
\hline
\end{array}
\]

(A) ethyl alcohol
(B) oil
(C) water
(D) glycerin

Ans: D

\[
\begin{align*}
p_0 &= 90 \text{ kPa} \\
p_1 &= p_0 + \gamma_1 h_1 = 90 \text{ kPa} + \left( 7.586 \text{ kPa/m} \right) (60 \text{ m}) = 545.16 \text{ kPa} \\
p_2 &= p_1 + \gamma_2 h_2 = 545.16 \text{ kPa} + \left( 8.825 \text{ kPa/m} \right) (10 \text{ m}) = 633.41 \text{ kPa} \\
p_3 &= p_2 + \gamma_3 h_3 = 633.41 \text{ kPa} + \left( 9.604 \text{ kPa/m} \right) (5 \text{ m}) = 681.43 \text{ kPa} \\
p_4 &= p_3 + \gamma_4 h_4 = 681.43 \text{ kPa} + \left( 12.125 \text{ kPa/m} \right) (5 \text{ m}) = 742 \text{ kPa}
\end{align*}
\]

Therefore, (D) is correct.
Example (FEIM):
The pressure at the bottom of a tank of water is measured with a mercury manometer. The height of the water is 3.0 m and the height of the mercury is 0.43 m. What is the gage pressure at the bottom of the tank?

\[ p_0 - p_a = \gamma_2 h_2 - \gamma_1 h_1 \quad [\text{U.S.}] \]

Ans: 27858 Pa
Fluid Mechanics 9-2b2
Fluid Statics

Example (FEIM):
The pressure at the bottom of a tank of water is measured with a mercury manometer. The height of the water is 3.0 m and the height of the mercury is 0.43 m. What is the gage pressure at the bottom of the tank?

From the table in the NCEES Handbook,
\[ \rho_{\text{mercury}} = 13560 \, \text{kg/m}^3, \rho_{\text{water}} = 997 \, \text{kg/m}^3 \]
\[ \Delta p = g(\rho_2 h_2 - \rho_1 h_1) \]
\[ = \left(9.81 \, \text{m/s}^2\right) \left(13560 \, \text{kg/m}^3 \cdot 0.43 \, \text{m}\right) - \left(997 \, \text{kg/m}^3 \cdot 3.0 \, \text{m}\right) \]
\[ = 27858 \, \text{Pa} \]

Fluid Mechanics 9-2c
Fluid Statics

Barometer

Atmospheric Pressure
\[ p_a - p_v = \rho gh \]

[SI] \[ 23.7 \, \text{a} \]
Example (FEIM):
The tank shown is filled with water. Find the force on 1 m width of the inclined portion.

\[
\bar{p} = \frac{1}{2} \rho g (h_1 + h_2) \quad [\text{SI}] \quad 23.10a
\]

\[
R = \bar{p}A
\]

\[
\text{Ans: } 90372 \text{ N}
\]
Fluid Mechanics 9-2e
Fluid Statics

Center of Pressure

\[ y^* = \frac{\rho g I_{x2} \sin \alpha}{p_c A} \quad [\text{SI}] \quad 22.17a \]

\[ z^* = \frac{\rho g I_{z2} \sin \alpha}{p_c A} \quad [\text{SI}] \quad 22.18a \]

If the surface is open to the atmosphere, then \( p_0 = 0 \) and

\[ p_c = \bar{p} = \rho g z_c \sin \alpha \quad [\text{SI}] \quad 23.19a \]

\[ y_{cp} - y_c = y^* = \frac{I_{y2}}{z_c A} \quad 23.20 \]

\[ z_{cp} - z_c = z^* = \frac{I_{y2}}{z_c A} \quad 23.21 \]

Fluid Mechanics 9-2f1
Fluid Statics

Example 1 (FEIM):
The tank shown is filled with water. At what depth does the resultant force act?

The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

Ans: 4.08 m
Fluid Mechanics

Example 1 (FEIM):
The tank shown is filled with water. At what depth does the resultant force act?

The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

\[ A = bh \]
\[ I_y = \frac{b^2h}{12} \]
\[ Z_c = \frac{4\text{ m}}{\sin 60^\circ} = 4.618\text{ m} \]

Using the moment of inertia for a rectangle given in the NCEES Handbook,

\[ z^* = \frac{I_y}{AZ_c} = \frac{b^2h}{12bhZ_c} = \frac{b^2}{12Z_c} \]
\[ = \frac{(2.31\text{ m})^2}{(12)(4.618\text{ m})} = 0.0963\text{ m} \]

\[ R_{\text{depth}} = (Z_c + z^*) \sin 60^\circ = (4.618\text{ m} + 0.0963\text{ m}) \sin 60^\circ = 4.08\text{ m} \]
Example 2 (FEIM):
The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m$^3$. The magnitude of the force $F$ per meter of width to keep the gate closed is most nearly

(A) 0 kN/m
(B) 24 kN/m
(C) 71 kN/m
(D) 370 kN/m

Ans: B

\[ \frac{R}{w} = \rho g z_{ave} \left( \frac{1600 \text{ kg}}{m^3} \right) \left( \frac{9.81 \text{ m/s}^2}{s} \right) \left( \frac{1}{2} \right) (3 \text{ m}) \]

\[ = 23544 \text{ Pa} \]

\[ R = \frac{p_{ave} h}{w} = (23544 \text{ Pa})(3 \text{ m}) = 70662 \text{ N/m} \]

\[ F + F_h = R \]

$R$ is one-third from the bottom (centroid of a triangle from the NCEES Handbook). Taking the moments about $R$,

\[ 2F = F_h \]

\[ F = \frac{1}{3} \left( \frac{R}{w} \right) = \frac{70667 \text{ N}}{3} = 23.6 \text{ kN/m} \]

Therefore, (B) is correct.
Fluid Mechanics

9-2h

Fluid Statics

Archimedes’ Principle and Buoyancy

- The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.
- A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

\[ F_{\text{buoyant}} = \gamma_{\text{water}} V_{\text{displaced}} \]

Fluid Mechanics

9-3a

Fluid Dynamics

Hydraulic Radius for Pipes

\[ R_H = \frac{\text{area in flow}}{\text{wetted perimeter}} \]

Example (FEIM):
A pipe has diameter of 6 m and carries water to a depth of 2 m. What is the hydraulic radius?

Ans: 1.12 m
Hydraulic Radius for Pipes

\[ R_H = \frac{\text{area in flow}}{\text{wetted perimeter}} \]

Example (FEIM):
A pipe has diameter of 6 m and carries water to a depth of 2 m. What is the hydraulic radius?

\[ r = 3 \text{ m} \]
\[ d = 2 \text{ m} \]
\[ \phi = (2 \text{ m})(\arccos((r - d) / r)) = (2 \text{ m})(\arccos \frac{1}{3}) = 2.46 \text{ radians} \]

(Caution! Degrees are very wrong here.)

\[ s = r\phi = (3 \text{ m})(2.46 \text{ radians}) = 7.38 \text{ m} \]
\[ A = \frac{1}{2}(r^2(\phi - \sin\phi)) = \frac{1}{2}(3 \text{ m})^2(2.46 \text{ radians} - \sin 2.46) = 8.235 \text{ m}^2 \]
\[ R_H = \frac{A}{s} = \frac{8.235 \text{ m}^2}{7.38 \text{ m}} = 1.12 \text{ m} \]

Continuity Equation

\[ \dot{m} = \rho A v = \rho Q \]
\[ \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \]

If the fluid is incompressible, then \( \rho_1 = \rho_2 \).

\[ Q = A_1 v_1 = A_2 v_2 \]
Fluid Mechanics 9-3c
Fluid Dynamics

Example (FEIM):

The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. The speed in the 130 mm pipe is most nearly

(A) 1 m/s
(B) 2 m/s
(C) 4 m/s
(D) 16 m/s

Ans: D

Therefore, (D) is correct.

\[ A_1 v_1 = A_2 v_2 \]

\[ A_1 = 4A_2 \]

so \( v_2 = 4v_1 = 4 \left( \frac{4 \text{ m/s}}{s} \right) = 16 \text{ m/s} \)
Fluid Mechanics 9-3d1
Fluid Dynamics

Bernoulli Equation

\[ \frac{p_1}{\gamma_1} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma_2} + \frac{v_2^2}{2g} + z_2 \]  [U.S.] 24.11b

• In the form of energy per unit mass:

\[ \frac{p_1}{\rho_1} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho_2} + \frac{v_2^2}{2} + gz_2 \]

Fluid Mechanics 9-3d2
Fluid Dynamics

Example (FEIM):
A pipe draws water from a reservoir and discharges it freely 30 m below the surface. The flow is frictionless. What is the total specific energy at an elevation of 15 m below the surface? What is the velocity at the discharge?

Ans: 294.3 J/kg
Ans: 24.3 m/s
Let the discharge level be defined as \( z = 0 \), so the energy is entirely potential energy at the surface.

\[
E_{\text{surface}} = z_{\text{surface}} \cdot g = (30 \, \text{m}) \left( \frac{9.81 \, \text{m}}{\text{s}^2} \right) = 294.3 \, \text{J/kg}
\]

(Note that \( \text{m}^2/\text{s}^2 \) is equivalent to \( \text{J/kg} \).)

The specific energy must be the same 15 m below the surface as at the surface.

\[
E_{15 \, \text{m}} = E_{\text{surface}} = 294.3 \, \text{J/kg}
\]

The energy at discharge is entirely kinetic, so

\[
E_{\text{discharge}} = 0 + 0 + \frac{1}{2}v^2
\]

\[
v = \sqrt{2 \left( \frac{294.3 \, \text{J}}{\text{kg}} \right)} = 24.3 \, \text{m/s}
\]

**Flow of a Real Fluid**

- Bernoulli equation + head loss due to friction

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f \quad [\text{U.S.}] \quad 24.12b
\]

\[
h_f = \frac{p_1 - p_2}{\gamma} \quad [\text{U.S.}] \quad 24.13b
\]

\( h_f \) is the head loss due to friction
Hydraulic Gradient

- The decrease in pressure head per unit length of pipe

Fluid Flow Distribution

If the flow is laminar (no turbulence) and the pipe is circular, then the velocity distribution is:

\[ v_r = v_{\text{max}} \left( 1 - \left( \frac{r}{R} \right)^2 \right) \]  

\( r \) = the distance from the center of the pipe  
\( v \) = the velocity at \( r \)  
\( R \) = the radius of the pipe  
\( v_{\text{max}} \) = the velocity at the center of the pipe
Fluid Mechanics
Fluid Dynamics

Reynolds Number
For a Newtonian fluid:

\[
Re = \frac{vD\rho}{\mu} \quad [\text{SI}] \quad 24.14g
\]

\[
Re = \frac{vD}{\nu} \quad 24.15
\]

\(D\) = hydraulic diameter = 4\(R_h\)
\(v\) = kinematic viscosity
\(\mu\) = dynamic viscosity

For a pseudoplastic or dilatant fluid:

\[
Re' = \frac{v^{3-n}D^n\rho}{K\left(\frac{3n+1}{4n}\right)^n} \quad 24.16
\]

Example (FEIM):
What is the Reynolds number for water flowing through an open channel 2 m wide when the flow is 1 m deep? The flow rate is 800 L/s. The kinematic viscosity is \(1.23 \times 10^{-6}\) m\(^2\)/s.

Ans: \(6.5 \times 10^5\)
**Fluid Mechanics**

**9-3h**

**Fluid Dynamics**

Example (FEIM):
What is the Reynolds number for water flowing through an open channel 2 m wide when the flow is 1 m deep? The flow rate is 800 L/s. The kinematic viscosity is $1.23 \times 10^{-6}$ m$^2$/s.

\[
D = 4R_p = \frac{4A}{P} = \frac{(4)(1 \text{ m})(2 \text{ m})}{2 \text{ m} + 1 \text{ m} + 1 \text{ m}} = 2 \text{ m}
\]

\[
v = \frac{Q}{A} = \frac{800 \text{ L}}{2 \text{ m}^2} = 0.4 \text{ m/s}
\]

\[
Re = \frac{vD}{\nu} = \frac{0.4 \text{ m/s}}{\frac{0.4 \text{ m}}{2 \text{ m}}} = 6.5 \times 10^5
\]

**9-4a**

**Head Loss in Conduits and Pipes**

**Darcy Equation**

- calculates friction head loss

\[
h_f = \frac{fLv^2}{2Dg}
\]

**Moody (Stanton) Diagram:**

![Moody Diagram](image-url)
Fluid Mechanics 9-4b

Head Loss in Conduits and Pipes

Minor Losses in Fittings, Contractions, and Expansions
- Bernoulli equation + loss due to fittings in the line and contractions or expansions in the flow area

\[ \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_L,\text{fitting} \]

\[ h_L,\text{fitting} = C \left( \frac{v^2}{2g} \right) \]

[US] 24.30b

24.31

Entrance and Exit Losses
- When entering or exiting a pipe, there will be pressure head loss described by the following loss coefficients:

<table>
<thead>
<tr>
<th>Sharp exit</th>
<th>Protruding pipe exit</th>
<th>Sharp entrance</th>
<th>Rounded entrance</th>
</tr>
</thead>
<tbody>
<tr>
<td>C = 1.0</td>
<td>C = 0.8</td>
<td>C = 0.5</td>
<td>C = 0.1</td>
</tr>
</tbody>
</table>

Fluid Mechanics 9-4b

Head Loss in Conduits and Pipes

Hydraulic grade line (HGL) and energy grade line (EGL) for a piping system.
Fluid Mechanics

Pump Power Equation

\[ P = \dot{W} = \frac{Q^2h}{\eta} \]

\[ = \frac{Q \rho gh}{\eta} \]

\[ = \frac{\dot{m} \rho gh}{\eta} \]

25.1

Fluid Mechanics

Impulse-Momentum Principle

\[ \sum F = Q_2 \rho v_2 - Q_1 \rho v_1 \quad [SI] \]

24.35a

Figure 24.7 Forces on a Pipe Bend

Pipe Bends, Enlargements, and Contractions

\[ -F_x = p_2 A_2 \cos \alpha - p_1 A_1 \]

\[ + Q_2 \rho (v_2 \cos \alpha - v_1) \quad [SI] \]

24.39a

\[ F_y = (p_2 A_2 + Q_2 \rho) \sin \alpha + m_{ext} g \quad [SI] \]

24.40a

F = force exerted by the bend on the fluid
Example (FEIM):
Water at 15.5°C, 275 kPa, and 997 kg/m³ enters a 0.3 m × 0.2 m reducing elbow at 3 m/s and is turned through 30°. The elevation of the water is increased by 1 m. What is the resultant force exerted on the water by the elbow? Ignore the weight of the water.

Ans: 13118 N
Impulse-Momentum Principle

Use the Bernoulli equation to calculate \( \rho_z \):

\[
\rho_z = \rho \left( -\frac{v^2}{2} + \frac{P_1}{\rho} + \frac{V}{2} + g(z_1 - z_2) \right)
\]

\[
= \left( \frac{997 \text{ kg/m}^3}{2} \right) \left( \frac{6.75 \text{ m/s}}{2} \right) + \frac{275000 \text{ Pa}}{\rho} + \frac{\left( \frac{3 \text{ m/s}}{2} \right)^2}{g(0 - 1 \text{ m})}
\]

\[
= 247000 \text{ Pa} \quad (247 \text{ kPa})
\]

\[ Q = \nu A \]

\[ F_x = -Q(\nu \cos \alpha - v_1) + P_A + P_2 \sin \alpha \]

\[
= -(3)(0.0707) \left( \frac{997 \text{ kg/m}^3}{2} \right) \left( \frac{6.75 \text{ m/s}}{2} \right) \cos 30^\circ - \frac{3 \text{ m/s}}{2} + (275 \times 10^3 \text{ Pa})(0.0707)
\]

\[
+ (247 \times 10^3 \text{ Pa})(0.0314 \text{ m}^2) \sin 30^\circ
\]

\[
= 256 \times 10^3 \text{ N}
\]

---

\[ F_y = Q(\nu \sin \alpha - 0) + P_2 \sin \alpha \]

\[
= (3)(0.0707) \left( \frac{997 \text{ kg/m}^3}{2} \right) \left( \frac{6.75 \text{ m/s}}{2} \right) \sin 30^\circ
\]

\[
+ (247 \times 10^3 \text{ Pa})(0.0314 \text{ m}^2) \sin 30^\circ
\]

\[
= 4592 \times 10^3 \text{ N}
\]

\[ R = \sqrt{F_x^2 + F_y^2} = \sqrt{(25600 \text{ kN})^2 + (4592 \text{ kN})^2} = 26008 \text{ kN} \]
Fluid Mechanics 9-7a
Impulse-Momentum Principle

Initial Jet Velocity: \[ v = \sqrt{2gh} \] 24.41

Jet Propulsion:
\[
F = \dot{m}(v_2 - v_1) = \dot{m}(v_2 - 0) = Q \rho v_2 = v_2 A_2 \rho v_2 = A_2 \rho v_2^2
= A_2 \rho \left( \sqrt{2gh} \right)^2 = 2gh A_2
= 2gh A_3 24.42
\]

Fluid Mechanics 9-7b1
Impulse-Momentum Principle

Fixed Blades

**Figure 24.9 Open Jet on a Stationary Blade**

\[ v_2 \]
\[ v_1 \]
\[ \alpha = \text{actual deflection angle} \]

\[ -F_x = Q \rho (v_2 \cos \alpha - v_1) \quad \text{[SI]} \quad 24.43a \]

\[ F_y = Q \rho v_2 \sin \alpha \quad \text{[SI]} \quad 24.44a \]
**Fluid Mechanics 9-7b2**

**Impulse-Momentum Principle**

Moving Blades

*Figure 24.10 Open Jet on a Moving Blade*

\[ F_x = -Q \rho (v_1 - v)(1 - \cos \alpha) \]  
\[ F_y = Q \rho (v_1 - v) \sin \alpha \]

**Fluid Mechanics 9-7c**

**Impulse-Momentum Principle**

Impulse Turbine

*Figure 24.11 Impulse Turbine*

The maximum power possible is the kinetic energy in the flow.

\[ P = Q \rho (v_1 - v)(1 - \cos \alpha) \]  
\[ P_{\text{max}} = \frac{Q (v_1^2)}{2} \]  
\[ P_{\text{max}} = \frac{Q \cos \alpha}{2} \]

The maximum power transferred to the turbine is the component in the direction of the flow.

\[ P_{\text{max}} = Q \rho \left( \frac{v_1^2}{2} \right) (1 - \cos \alpha) \]  
\[ P_{\text{max}} = \frac{Q \cos \alpha}{2} \]
1) The flow divides as to make the head loss in each branch the same.
\[ h_{f,A} = h_{f,B} \]
\[ \frac{f_A L}{D_A} v_A^2 = \frac{f_B L}{D_B} v_B^2 \]

2) The head loss between the two junctions is the same as the head loss in each branch.
\[ h_{f,1} - h_{f,A} - h_{f,B} \]

3) The total flow rate is the sum of the flow rate in the two branches.
\[ \frac{1}{2} D_1^2 v_1 - \frac{1}{2} D_A^2 v_A + \frac{1}{2} D_B^2 v_B - \frac{1}{2} D_2^2 v_2 \]

- Mass must be conserved.
\[ D^2 v = D_A^2 v_A + D_B^2 v_B \]

**Fluid Mechanics**

**Multipath Pipelines**

In an ideal gas:
\[ c = \sqrt{kRT} \]

Mach Number:
\[ M = \frac{v}{c} \]

Example (FEIM):
What is the speed of sound in air at a temperature of 339K? The heat capacity ratio is \( k = 1.4 \).
\[ c = \sqrt{kRT} = \sqrt{(1.4)\left(286.7 \frac{m^2}{s^2 \cdot K}\right)(339K)} = 369 \text{ m/s} \]

Ans: 369 m/s
Pitot Tube – measures flow velocity

\[ h_s = \frac{p_0}{\gamma} \]

- The static pressure of the fluid at the depth of the pitot tube \((p_0)\) must be known. For incompressible fluids and compressible fluids with \(M \leq 0.3\),

\[ v = \sqrt{\frac{2(p_0 - p_a)}{\rho}} \]  

[SI] 25.11b

Example (FEIM):
Air has a static pressure of 68.95 kPa and a density 1.2 kg/m³. A pitot tube indicates 0.52 m of mercury. Losses are insignificant. What is the velocity of the flow?

Ans: 26.8 m/s
Fluid Mechanics
Fluid Measurements

Example (FEIM):
Air has a static pressure of 68.95 kPa and a density 1.2 kg/m³. A pitot tube indicates 0.52 m of mercury. Losses are insignificant. What is the velocity of the flow?

\[ p_0 = \rho_{\text{mercury}} \cdot gh = \left( 13560 \, \frac{\text{kg}}{\text{m}^3} \right) \left( 9.81 \, \frac{\text{m}}{\text{s}^2} \right) (0.52 \, \text{m}) = 69380 \, \text{Pa} \]

\[ v = \sqrt{\frac{2(p_0 - p_s)}{\rho}} = \sqrt{\frac{2(69380 \, \text{Pa} - 68950 \, \text{Pa})}{1.2 \, \frac{\text{kg}}{\text{m}^3}}} = 26.8 \, \text{m/s} \]

---

Fluid Mechanics
Fluid Measurements

Venturi Meters – measures the flow rate in a pipe system
- The changes in pressure and elevation determine the flow rate. In this diagram, \( z_1 = z_2 \), so there is no change in height.

Figure 25.2 Venturi Meter with Differential Manometer

\[ Q = \left( \frac{C_1 A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)} \]

[US] 25.14b
Example (FEIM):
Pressure gauges in a venturi meter read 200 kPa at a 0.3 m diameter and 150 kPa at a 0.1 m diameter. What is the mass flow rate? There is no change in elevation through the venturi meter.
Assume $C_v = 1$ and $\rho = 1000 \text{ kg/m}^3$.

(A) 52 kg/s
(B) 61 kg/s
(C) 65 kg/s
(D) 79 kg/s

$Q = \left( \frac{C_v A_2}{A_1} \right) \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)}$

Ans: D

Therefore, (D) is correct.
Fluid Mechanics 9-10e
Fluid Measurements

Orifaces

\[ Q = CA \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)} \]  

[U.S.] 25.17b

Fluid Mechanics 9-10f
Fluid Measurements

Submerged Orifice  

\[ Q = A \sqrt{2g(h_1 - h_2)} \]  

25.18

\[ C - C_c \]  

25.19

and \( C_c \) = coefficient of contraction

Orifice Discharging Freely into Atmosphere

\[ Q = CV\sqrt{2gh} \]  

\[ v = \sqrt{2gh} \]  

25.17a

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Drag Coefficients for Spheres and Circular Flat Disks

\[ F_D = \frac{C_D \pi D v^2}{2} \quad \text{[SI]} \quad 24.55 \]

Fluid Mechanics 9-10g
Fluid Measurements