FE Thermodynamics Review

Dr. David R. Muñoz
Definitions

- System – collection of matter which we are interested in studying.
- Surroundings – everything outside of the system.
- Universe – system + surroundings.
- Pure substance – its chemical composition remains the same regardless of phase.
- Intensive property – thermodynamic property independent of the system mass.
- Independent property – a stand alone property that can be used to identify the state of a system.
Definitions (cont.)

• State postulate for a pure substance – takes two independent intensive properties to fix its state.

• Closed system – mass does not cross the system boundaries, but heat and work can cross the system boundaries

• Open system – mass crosses the system boundaries, heat and work can also cross the system boundaries
  - Steady flow – open system with no mass accumulation
  - Unsteady flow - open system with mass accumulation

• Sign convention: Heat into system is positive. Work out of system is positive.
General Exam Taking Philosophy

• You should prepare and work to get the correct answer whenever possible.

• However, on those problems you get stuck on, remember that wrong answers are no worse than no answer.

• Therefore, first try to eliminate unreasonable answers to improve the odds of guessing right.

• Then make your best guess.

• If you don’t have time to eliminate wrong answers, pick your favorite letter and guess.
Problem 1. What is the mass of air contained in a room 20m x 40m x 3m at standard conditions?

• Standard conditions may vary depending on what organization is defining them. It is generally,
• $P = 1 \text{ atm} = 101 \text{ kPa}$ at sea level,
• $T = 25^\circ \text{C} = 298 \text{K}$
• Assume: air is an ideal gas
• You are given the universal gas constant as $R_u = 8.314 \text{ kJ/kmol \ K}$, where $R = R_u / M$,
  $M =$ molecular weight for the particular gas of interest.
Problem 1 (cont.)

Ideal gas Law: PV = mRT
m = PV/RT

R = [8.314 kJ/kmol K]/[29 kg/kmol] = 0.287 kJ/kg K

V = [20 x 40 x 3] m³ = 2400 m³

Therefore,

m = [(101 kPa)(2400 m³)]/[(0.287 kJ/kg K (298 K)]
m = 2834 kg
Problem 2 – Find the volume occupied by 20 kg of steam at 0.4 MPa, 400°C

• Use steam tables provided in your FE booklet
• At this point you do not know whether to use the saturated mixture table or the superheated steam table, so that must first be determined.
• Recall the vapor dome associated with the saturation conditions for water (see next slide)
Temperature vs. volume and Pressure vs. volume diagrams
Saturation Table

Problem 2 (cont.)

- Temperature column only goes to 374°C, which is the critical point for water (apex of the vapor dome).

- Therefore, you must use the superheated vapor table for Problem 2.
Superheated Vapor Table

Problem 2 (cont.)

At 0.4 MPa, 400°C

\[ \nu = 0.7726 \text{ m}^3/\text{kg} \]

\[ V = \nu m = (20) \text{kg} \times (0.7726) \text{ m}^3/\text{kg} \]

\[ V = 15.45 \text{ m}^3 \]
Problem 3 – How much heat must be added to 2 kg of steam initially at $T_1 = 300^\circ C$, contained within a rigid volume to increase the pressure from 0.2 MPa to 0.4 MPa?

Solution:
Assume: Closed System [No mass crosses the Control Surface (CS) but heat or work can cross the CS.]

Rigid tank means the volume does not change: $V_1 = V_2$, $v_1 = v_2$

$1^{st}$ Law (closed system)

$Q = \Delta U = m(\Delta u)$

At $T_1 = 300^\circ C$, $P_1 = 0.2$ MPa (using the superheat table for water)

$v_1 = 1.3162 \text{ m}^3/\text{kg}$; $u_1 = 2808.6 \text{ kJ/kg}$

Since $v_1 = v_2$, at $P_2 = 0.4$ MPa, $v_2 = 1.3162 \text{ m}^3/\text{kg}$

$u_2 = 3793.2 \text{ kJ/kg}$ (by interpolation)

Therefore: $Q = (2) \times (3793.2 - 2808.6) \text{ kJ/kg}$

$Q = 1969 \text{ kJ}$
Linear Interpolation for Problem 3

Since it takes two independent intensive properties to fix the state of a pure substance, in the case of problem 3, the state 2 is fixed since we know the pressure and specific volume. However, since the tables are not tabulated in round numbers of specific volume, we must interpolate between two values that bracket the specific volume given.

Looking at the superheated water table, we notice that for a pressure of 0.4MPa, the specific volume is bracketed between the temperatures of 800°C and 900°C. We note the internal energies for these temperatures. The next step is to set up a linear relationship between these values and solve for the desired internal energy corresponding to the known specific volume.

\[
\frac{v_2 - v_A}{v_B - v_A} = \frac{u_2 - u_A}{u_B - u_A}
\]

\[
u_2 = u_A + \left(\frac{v_2 - v_A}{v_B - v_A}\right)(u_B - u_A) = (3662.4)\text{kJ/kg} + \left(\frac{1.3162 - 1.2372}{1.3529 - 1.2372}\right)(3853.9 - 3662.4)\text{kJ/kg}
\]

\[
u_2 = 3793.2 \text{ kJ/kg}
\]
Problem 4 – How much heat is needed to completely vaporize 100 kg of water from temperature $T_1 = 20^\circ C$ if the pressure is maintained at a constant $P = 200$ kPa absolute?

Solution:
The process is shown in the T-v diagram drawn on the left. State 1 begins as a compressed liquid and state 2 is shown as a saturated vapor. The process follows a line of constant pressure (as indicated in the problem statement). Since we do not have access to a compressed liquid table for this exam, you have to make an assumption regarding the property selection for state 1. Since the water is initially in the liquid state, we consider it to be largely incompressible. Therefore, we cannot add much internal energy to the water by compressing it. However, a small temperature increase in the water does contribute to the water internal energy. Therefore, we will assume that due to liquid incompressibility, we can use the property of the saturated liquid, evaluated at this initial temperature, to represent state 1 of the liquid.
Problem 4 (cont.)

Solution (cont.)
1\textsuperscript{st} Law (closed system)
Q = \Delta U = m(\Delta u)

Where \( u_1 = u_f \) (at 20\textdegree C); \( u_1 = 83.95 \text{ kJ/kg} \) (saturation table)

\( u_2 = u_g \) (at 200 kPa); \( u_2 = 2529.5 \text{ kJ/kg} \) (superheat table)

\[
Q = (100)(2529.5 - 83.95) \text{ kJ/kg}
\]

\( Q = 244,555 \text{ kJ} \)
**Problem 5** – Calculate the work done by a piston contained within a cylinder with air if 2 m$^3$ is tripled while the temperature is maintained at a constant $T = 30^\circ\text{C}$. The initial pressure is $P_1 = 400$ kPa absolute.

**Solution:**
Assume air is an ideal gas. $PV = mRT$

$V_1 = 2 \text{ m}^3$; $V_2 = 3V_1 = 6 \text{ m}^3$

$P = \frac{mRT}{V}$

Work,

$$W = \int_{V_1}^{V_2} PdV = mRT \int_{V_1}^{V_2} \frac{dV}{V} = mRT \left[ \ln \frac{V_2}{V_1} \right]$$

but

$$m = \frac{P_1 V_2}{RT_2}$$

Therefore,

$$W = \frac{P_1 V_2}{RT_2} RT \left[ \ln \frac{V_2}{V_1} \right] = P_1 V_1 \left[ \ln \frac{V_2}{V_1} \right] = (400) \text{kPa}(2)\text{m}^3 \ln \frac{6}{2} = 878.9 \text{kJ}$$
Problem 6 – How much work is necessary to compress air in an insulated cylinder from 0.2 m³ to 0.01 m³? Use $T_1 = 20^\circ$C and $P_1 = 100$ kPa absolute.

Solution:
Assume:
- Ideal gas
- Isentropic process
- Constant $c_v$
- Adiabatic process (insulated container)

1\textsuperscript{st} Law (closed system)
$\varnothing = \Delta U + W = m(\Delta u) + W$

$W = -m \ c_v \ (T_2 - T_1) \ ; \ W/m = w = c_v \ (T_2 - T_1)$

Isentropic relationships
$T_1 \ v_1^{k-1} = T_2 \ v_2^{k-1}$ where $k = 1.4$ (for air)

$T_2 = T_1 \ \{v_1/v_2\}^{k-1} = T_1 \ \{V_1/V_2\}^{k-1} = (293)K \ {(0.2)/(0.01)}^{1.4-1} = 971K$

$w = -(0.719) \ \text{kJ/kg} \ \times \ (971 - 293) \ \text{K} = -487 \ \text{kJ/kg}$
Problem 7 – A 10 cm thick slab of wood \( (k_{\text{wood}} = 0.2 \text{ W/m K}) \) is 3 m high and 10 m long. Calculate the heat transfer rate if the temperature is 25\(^\circ\)C on the inside and -20\(^\circ\)C on the outside. Neglect convection.

\[
q_{\text{in}} = 25^\circ\text{C} \quad q_{\text{out}} = -20^\circ\text{C}
\]

**Solution:**

Assume: conduction heat transfer only

\[
A = L W = (10) \text{ m} (3) \text{ m} = 30 \text{ m}^2
\]

\[
q = -k A \left( \frac{dT}{dx} \right) \sim - k A \left( \frac{\Delta T}{\Delta x} \right)
\]

\[
q = -(0.2) \text{ W/m K} \times (30) \text{ m}^2 \times (-20 - 25) \text{ °C}/(0.1) \text{ m}
\]

\[
q = 270 \text{ W}
\]
Problem 8 – The surface of the glass in a 1.2 m x 0.8 m skylight is maintained at 20°C and the outside air temperature is -20°C. Estimate the heat loss rate by convection when the convective coefficient is \( h = 12 \text{ W/m}^2 \text{ K} \).

Solution:
Assume convective heat transfer only

\[
q = h A (T_s - T_\infty)
\]

\[
q = (12) \text{ W/m}^2 \text{ K} (1.2 \text{ m}) (0.8 \text{ m}) (20 - (-20)) \text{ °C}
\]

\[
q = 460.8 \text{ W}
\]
**Problem 9** – A 2 cm diameter heating element located within an oven is maintained at 1000°C and the oven walls are at 500°C. If the element emissivity is 0.85, estimate the rate of heat loss from the 2 m long element.

**Solution:**
Assume radiation heat transfer only
- Use absolute temperatures,
- Treat the oven walls as large surroundings.

\[ T_s = 1000°C = 1273K ; \quad T_{sur} = 500°C = 773K \]

\[ q = \varepsilon \sigma A \left( T_s^4 - T_{sur}^4 \right) ; \quad \text{where} \quad A = \pi dL \text{ (“circumferencial” area)} \]

\[ q = (0.85) \left( 5.67 \times 10^{-8} \right) \frac{W}{\mu^2 K^4} (\pi) (0.02) \pi (2) \left[ (1273)^4 - (773)^4 \right] K^4 \]

\[ q = 13,742 \text{ W} = 13.7 \text{ kW} \]
Problem 10 – Refrigerant R134a expands through a valve from a pressure of 800kPa to a pressure of 100kPa. What is the final quality?

Solution:
There is no table of thermodynamic data for 134a contained within your FE book. Therefore you must use the Pressure – Enthalpy diagram provided (see subsequent pages). This diagram is much like the Temperature – volume diagram for water in that the apex of the dome is the critical point (CP). The solid line corresponding to saturated liquid is on the left and that associated with the saturated vapor is on the right.

1st Law (Steady Flow, expansion valve),
\[ h_1 = h_2 \]

We commonly use expansion valves in vapor compression refrigerators to move from the high pressure and temperature portion of the cycle to the low pressure and temperature portion of the cycle. State 1 is typically considered to be in a saturated liquid state. Please see FE book for list of commonly used steady flow devices and their associated modeling equations.
Problem 10 (cont.)

Solution (cont.)
\[ h_1 = h_f = 244 \text{ kJ/kg} = h_2 \]

From the P vs. h plot (next page), find the quality (these are the curves emanating from the critical point)
\[ x = 0.37 \]
Pressure – Enthalpy diagram for R-134a
Problem 11 – What is the minimum pump power required to increase the pressure of water from 2 kPa to 6 MPa with a mass flowrate of 10 kg/s?

Solution:

1st Law (pumps, isentropic or reversible steady flow)

\[ w = - \int_{1}^{2} v dP = -v_f (P_2 - P_1) \]

Use the saturation table for water to find \( v_f \).

\[ v_f = 0.001002 \text{ m}^3/\text{kg} \]

\[ w = -(0.001002) \text{ m}^3/\text{kg} \times (6000 - 2) \text{ kPa} \]

\[ w = -6.0 \text{ kJ/kg} \]

\[ W = \dot{m} w = \frac{(10) \text{ kg}}{s} \times \frac{(-6.0) \text{ kJ}}{\text{kg}} = -60 \text{ kW} \]
Problem 12 – A refrigeration system using R-134a operates between -20\(^\circ\)C and 40\(^\circ\)C. What is the maximum coefficient of performance (COP)?

Solution:

The maximum COP is achieved in a Carnot cycle. For a Carnot cycle,

\[
\frac{T_H}{T_L} = \frac{Q_H}{Q_L}
\]

Remember to use absolute temperatures.

\[
COP = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{1}{\frac{40 + 273}{-20 + 273} - 1} = 4.22
\]
Problem 13 – A heat pump delivers 20,000 kJ/hr with q 1.39 kW electrical input. Calculate the COP.

Solution:
A heat pump is like a refrigerator. It different in that includes a special 4-way valve that allows the user to switch the functions of the heat exchangers for the heating season (winter) and the cooling season (summer). Unless otherwise stated in the problem statement, the heat pump is usually considered to be in the heating mode for these analyses. In the heating mode, that which we desire is \( Q_H \). Therefore, the COP is defined as follows.

\[
COP = \frac{Q_H}{W} = \frac{(20,000)\text{kJ/hr}}{(1.39)\text{kW}(3600)\text{sec/hr}} = 4.0
\]
Problem 14 – A thermometer with a wet cloth attached to its bulb reads 20°C when air is blowing around it. If the air has a dry bulb temperature of 33°C, what is the relative humidity and dew point temperature? How much water could be condensed out of a 100 m³ volume?

Solution:
You will need to use the psychrometric chart provided in your FE book. It has been copied on the following pages.

A wet bulb temperature of 20°C is provided in the problem statement. The lines corresponding to the wet bulb temperature on the psychrometric chart are dashed and move from the upper left to the lower right (locate them on the chart). The dry bulb temperature is the abscissa of the psychrometric chart. The intersection between the 20°C wet bulb and the 33°C dry bulb temperature corresponds to a relative humidity of 30% (the relative humidity curves move from the lower left to the upper right on the chart).

The dew point temperature is that temperature when water begins to condense. Since the ordinate of the psychrometric chart is the humidity ratio (grams moisture/kg dry air), which is proportional to the absolute humidity. The dew point temperature is the dry bulb temperature that corresponds to the intersection of a horizontal line drawn through state 1 and the saturation line (corresponding to 100% relative humidity). This is a dew point temperature of ~13°C.
Problem 14 (cont.)

Solution (cont.)
Condensate is taken from the air when it is cooled to below the dew point temperature. The amount of condensate that can be removed corresponds to that associated with the difference in the moisture content at the dew point temperature and that at 0°C. Below 0°C, the phase will be frost or ice, corresponding to a crystallization (vapor – solid) phase change process.

Therefore,

\[
\text{Mass of Condensate} = m(\omega_2 - \omega_1),
\]

where \( m \) is the mass of dry air and \( \omega \) is the humidity ratio (taken from psychrometric chart)

Assuming dry air is an ideal gas

\[
m = \frac{PV}{RT} = \frac{[(101)\text{kPa} \times (100)\text{m}^3]}{[(287)\text{kJ/kg K} \times (33+273)\text{K}]} = 115\text{kg}
\]

\[
\text{Mass of condensate} = (115)\text{kg dry air (9.3 – 3.7) gm moisture/kg dry air} = 644\text{ gm moisture}
\]
Psychrometric Chart
**Problem 15** – It is desired to condition $35^\circ C$, 85% relative humidity air to $24^\circ C$, 50% relative humidity. If $100 \text{ m}^3/\text{min}$ air is to be conditioned, how much energy is required in the cooling process and how much in the heating process?

**Solution:**

One way to remove the moisture is to first cool the air to the saturation condition (100% relative humidity line) and continue the cooling process to remove moisture until the desired humidity ratio is achieved. Therefore, the engineer must locate the initial (1) and final (4) state points first and then work with the psychrometric chart (as shown on the following pages) to determine the energy requirements.

From psychrometric chart $h_1 = 110 \text{ kJ/kg dry air}$, $h_2 = 106 \text{ kJ/kg dry air}$ (see enthalpy on upper left). This is the sensible (no phase change) cooling part.

\[
Q_c = \dot{m}(h_2 - h_1) = \rho \dot{V}(h_2 - h_1) = \frac{P}{RT} \dot{V}(h_2 - h_1)
\]

\[
Q_c = \frac{(101) \text{kPA}}{(0.287) \text{ kJ/kg K}} \frac{(100) \text{ m}^3/\text{min}}{(35 + 273) K} \frac{(100) \text{ m}^3/\text{min}}{(60) \text{ sec/min}} [(106) - (110)] \text{kJ/kg} = -7.62 \text{kW}
\]
Problem 15 (cont.)

Solution (cont.)
Sensible heating
From the chart $h_3 = 37 \text{ kJ/kg dry air}$, $h_4 = 48 \text{ kJ/kg dry air}$

$$
Q_h = \dot{m}(h_4 - h_3) = \rho \dot{V}(h_4 - h_3) = \frac{P}{RT} \dot{V}(h_4 - h_3)
$$

$$
Q_h = \frac{(101) \text{kPA}}{(0.287) \text{kJ/kg K}} \frac{(100) \text{m}^3/\text{min}}{(35 + 273) \text{K}} \frac{(37) \text{kJ/kg}}{(60) \text{sec/min}} \dot{V} = 20.95 \text{kW}
$$

Latent and sensible cooling

$$
Q_{c(l+s)} = \dot{m}(h_3 - h_2) = \rho \dot{V}(h_3 - h_2) = \frac{P}{RT} \dot{V}(h_3 - h_2)
$$

$$
Q_{c(l+s)} = \frac{(101) \text{kPA}}{(0.287) \text{kJ/kg K}} \frac{(100) \text{m}^3/\text{min}}{(35 + 273) \text{K}} \frac{[(48) - (37)] \text{kJ/kg}}{(60) \text{sec/min}} \dot{V} = -131.4 \text{kW}
$$

Total Cooling = -7.62 - 131.4 kW = -139 kW; Total Heating = 21 kW