Bohr Model of the Atom

Niels Bohr
1885-1962  Denmark, US
1922 Nobel Prize in Physics
Niels Bohr’s postulates

- Existence of “stationary” states for the orbiting electrons

- Transitions between stationary states:
  \[ E = E_1 - E_2 = h\nu \]

- Classical laws of physics do not apply to transitions between stationary states

- The electron can only exist in orbits for which its angular momentum is given by:
  \[ L = \lvert \mathbf{r} \times \mathbf{p} \rvert = mv r = \frac{nh}{2\pi} \text{ with } n=1,2,3,\ldots \]
3) What is the speed (v/c) of an electron in the first three Bohr orbits of the H atom? (Ch4 #22)

4) A hydrogen atom in an excited state absorbs a photon of wavelength 434 nm. What were the initial and final states of the hydrogen atom? (ch 4# 23)

5) If a hydrogen atom is initially in the first excited state, what is the longest wavelength of light it will absorb? What is the shortest wavelength of light it will absorb?

6) What is the calculated binding energy of the electron in the ground state of (a) deuterium, (b) He+, (c) Li++? (ch4# 25)

7) Hydrogen atoms in highly excited states with a quantum number as large as n=732 have been detected in interstellar space by radio astronomers. What is the orbital radius of the electron in such an atom? What is the energy of the electron?
Spectral Lines of Hydrogen

Energy (eV) | Binding energy (eV)
---|---
0 | 0
-0.38 | 0.38
-0.54 | 0.54
-0.85 | 0.85
-1.51 | 1.51

n > 1, I = 1 & n = 2, 3, 4, ... Lyman Series
I = 2 & n = 3, 4, 5, ... Balmer Series
I = 3 & n = 4, 5, 6, ... Paschen Series

By 1913, some of these lines were not observed. Still they were predicted by Bohr's model.
• If an electron in a hydrogen atom makes a transition from some state to a lower state do the following increase or decrease
  – Kinetic energy
  – Potential energy
  – Orbital angular momentum
• What energy is required to ionize a hydrogen atom with an electron in the first excited state? (remove the electron from the atom?)
Planetary Model

- Force Applied to the electron:
  \[ \vec{F}_e = \frac{-1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \hat{e}_r = \frac{mv^2}{r} \]

- One deduces: \( v = \frac{e}{(\sqrt{4\pi\varepsilon_0 m}r)} \)

- Total energy with \( K = (1/2)mv^2 \):
  \[ E = K + V = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r} \]
Failure of the classical (planetary) atomic model

- Atom (neutral) = nucleus (+q) + q electrons

- Assuming the Hydrogen atom:
  - The electron is attracted by the nucleus
  - Even in circular motion around the nucleus, the electron loses energy:
    - Radial acceleration: $a_r = \frac{v^2}{R}$
    - Classical e.m. theory: an accelerating charge continuously radiates energy, $r$ decreases...

The electron would eventually crash into the nucleus
Failure of the planetary model

- Doomed because the electron radiates energy, while orbiting around the nucleus

- But:
  - There is some “truth” to it, since Rutherford was successful in describing the scattering experiment

- In 1913, Niels Bohr 1885-1962 postulates that the electrons may be in stable (non-radiating) circular orbits, called stationary orbits
Bohr Radius

- From Bohr’s postulate: $v = \frac{n\hbar}{mr}$
- Since: $v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$ [Planetary Model]
- One can deduce the diameter of the hydrogen atom for stationary states:
  $$r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{me^2} = n^2 a_0 \quad \text{with } a_0, \text{ the Bohr radius}$$
Energy Levels of the Hydrogen atom

- Energy of the stationary states:

\[ E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv -\frac{E_0}{n^2} \]

with \( E_0 = -13.6 \text{ eV} \)

Emission of light: \( h\nu = E_{n_u} - E_{n_l} \)

Using: \( 1/\lambda = \nu/c \),

\[ \frac{1}{\lambda} = \frac{E_u - E_\ell}{hc} = R_\infty \left( \frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \]

\( R_\infty \), the Rydberg constant for Hydrogen
Spectral Lines of Hydrogen

\[
\begin{align*}
\text{Energy (eV)} & \quad \text{Binding energy (eV)} \\
0 & \quad 0 \\
-0.38 & \quad -0.38 \\
-0.54 & \quad -0.54 \\
-0.85 & \quad -0.85 \\
-1.51 & \quad -1.51 \\
\vdots & \quad \vdots \\
-3.40 & \quad 3.40 \\
\vdots & \quad \vdots \\
-13.6 & \quad 13.6
\end{align*}
\]

By 1913, some of these lines were not observed. Still they were predicted by Bohr’s model.

- For \( n > l \), \( l=1 \) and \( n=2,3,4,\ldots \) Lyman Series
- For \( l=2 \) and \( n=3,4,5,\ldots \) Balmer Series
- For \( l=3 \) and \( n=4,5,6,\ldots \) Paschen Series
Quantum to Classical Mechanics?

- The Correspondance Principle (Bohr):
  
  In the limits where classical and quantum theories should agree, the quantum theory must reduce to the classical result.
Follow up on Born & Examples
Example w/ applet on Rutherford scattering

Thompson model was shown to be flawed
Atoms are mostly empty space, with dense positive nucleus in the center

\[ F = \frac{k q_1 q_2}{r^2} \]

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = -\frac{Z e^2}{r} \]

\[ F = m \frac{dv}{dt} = m\left(\frac{v^2}{r}\right) = -\frac{Z e^2}{4\pi\varepsilon_0} \frac{1}{r^2} \]

Electron is slow enough that we don't need to use special relativity

\[ E = kE + U \]

**KINETIC** **POTENTIAL**

\[ \frac{dv}{dt} = \frac{v^2}{r} \quad \text{NOT DERIVED HERE} \]

\[ F = ma = \frac{v^2}{r} \cdot m \]

Consider gravity which is also a \( \frac{1}{r^2} \) force

\[ F = \frac{G m M}{r^2} \]

\[ \text{A) Drop a ball from a crane. It falls straight down} \]

\[ \text{B) Throw a ball with some velocity. It makes a parabola} \]

\[ \text{C) Throw it really hard, parabola is "wider" ball drops further} \]

\[ \text{G) Throw it harder enough + it goes into orbit!} \]

What is acceleration? magnitude of \( v \) is constant, direction changes \( \frac{dv}{dt} \neq 0 \)
So
\[ E = kE + U \]
\[ \frac{1}{2}mv^2 + U \]

\[ \frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r^2} \]

\[ \frac{1}{2} \times \frac{mv^2}{r} = \frac{1}{2} \times \frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r^2} \]

\[ \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0} \frac{1}{r} = kE \]

So now we need to figure out what potential energy "U" is for a \( 1/r^2 \) electric force.

This matters because total energy is \( E = kE + U \).

In quantum mechanics we will encounter \( U \) in the Schrödinger equation.

Different systems (Hydrogen atom, simple harmonic oscillator) have different expressions for \( U \).
**SAME SIGN CHARGES**
- **Repulsive Force**
- Bring charges closer requires energy to do this
- Potential energy increases
- Must do work (add energy) to bring charges together

**OPPOSITE SIGN CHARGES**
- **Attractive Force**
- When $r$ gets smaller, potential energy decreases
- Must do work (add energy) to pull them apart

**Potential Energy Equation**

$$U = \frac{q'q}{4\pi\varepsilon_0} \frac{1}{r}$$

For hydrogen atom $q' = +Ze$, $q = -e$

$$U = -\frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r}$$

$$U(r) = \int_{\infty}^{r} F(r') \, dr'$$
SO PUT IT ALL TOGETHER

\[ E = KE + U \]

\[ = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 r} \]

\[ E_e = -\frac{e^2}{8\pi\varepsilon_0 r} \]

BUT WE HAVE A PROBLEM . . . . . .

ANTENNA

oscillating charge radiates
(accelerating)
energy, think radio antenna

OUR ELECTRON IS ACCELERATING.
CLASSICALLY IT WILL RADIATE ENERGY TOO

ATOM WILL COLLAPSE \( \sim 10^{-10} \) SECONDS!

Q: WHY DOESN'T OUR SOLAR SYSTEM DO THE SAME THING?

BOTH FORCES ARE \( r^{-2} \)?

\[ F_{\text{em}} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2} \]

\[ F_{\text{grav}} = \frac{G Mm}{r^2} \]

ANS

Gravity much weaker, does radiate Gravity waves
but energy loss is tiny \( \sim \) trillions of years to change orbit
So somehow we need a model of the atom where it does not collapse.

Recall Planck invoked quantization of energy to evade a different problem (UV catastrophe) of radiated energy.

This time Niels Bohr invoked quantization of angular momentum.

\[ L = n \hbar \]

\[ L = r \cdot m \cdot v = n \frac{\hbar}{2\pi} = n \hbar \quad \hbar = \frac{\hbar}{2\pi} \]

\[ n = 1, 2, 3, \ldots \]

**Quantization of Angular Momentum**

So it \( L \) is quantized, \( E \) will be quantized, \( r \) will be quantized (under this model).

\[ E = \frac{1}{2} m v^2 = \frac{1}{2} \frac{e^2}{4\pi\varepsilon_0} \frac{1}{r} \]

\[ \frac{1}{2} m \left( \frac{n\hbar}{2\pi} \right)^2 = \frac{1}{2} \frac{e^2}{4\pi\varepsilon_0} \frac{1}{r} \]

\[ \frac{1}{2} \frac{n^2\hbar^2}{m} \frac{1}{r^2} = \frac{1}{2} \frac{e^2}{4\pi\varepsilon_0} \frac{1}{r} \]

\[ n^2 \frac{4\pi\varepsilon_0 \hbar^2}{m e^2} = r = n^2 a_0 \quad a_0 = 0.0529 \text{ nm} \]

**Bohr Radius**

\[ r = a_0 \quad 4a_0 \quad 9a_0 \quad \ldots \]

\[ n = 1 \quad n = 2 \quad n = 3 \]
\[ E_n = -\frac{e^2}{8\pi\varepsilon_0 r} \quad r = n^2 a_0 \]

\[ = -\frac{m_e e^4}{2(4\pi\varepsilon_0 \hbar)^2} \frac{1}{n^2} \quad \text{PLUS THIS IN} \]

\[ \frac{e^2}{4\pi\varepsilon_0} = 1.44 \text{ eV nm} \]

\[ m_e = 511,000 \text{ eV/c}^2 \]

\[ \hbar c = 12.40 \text{ eV nm} \]

\[ = -\frac{m_e (2\pi)^2 e^2}{2 (\hbar c)^2} \left( \frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{1}{n^2} \]

\[ = -\left( \frac{511,000 \text{ eV}}{c^2} \right) \frac{2\pi}{2} \left( \frac{1}{\hbar c} \right)^2 \left( \frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{1}{n^2} \]

\[ = -511,000 \text{ eV} \left( \frac{2\pi}{2} \right)^2 \left( \frac{1}{12.40 \text{ eV nm}} \right)^2 \frac{1}{n^2} \]

\[ E_{en} = -13.6 \text{ eV} \frac{1}{n^2} \]

Hydrogen Atom, Bohr Model

\[ E_e = -\frac{m_e e^4}{2(4\pi\varepsilon_0 \hbar)^2} \frac{1}{n^2} \]

\[ \frac{\hbar c}{\lambda} = \frac{\hbar 2\pi c}{\lambda} = -\frac{m_e e^4}{2(4\pi\varepsilon_0 \hbar)^2} \frac{1}{n^2} \]

\[ \frac{1}{\lambda} = \frac{m_e e^4}{64\pi^2\varepsilon_0^2 \hbar^2 c^3} \frac{1}{n^2} \]

\[ R_n \text{ Rydberg} = \frac{1}{91,128 \text{ nm}} \]
\[ \Delta E = E_{\text{INIT}} - E_{\text{FINAL}} \]

Suppose \( E_{\text{INIT}} = -13.6 \text{eV}(\frac{1}{2})^2 \), \( E_{\text{FINAL}} = -13.6 \text{eV}(\frac{1}{4})^2 \)

\[ \Delta E = -13.6 \text{eV}(\frac{1}{2})^2 - (-13.6 \text{eV}(\frac{1}{4})^2) \]

\[ = 13.6 \text{eV} - \frac{13.6}{4} \text{eV} \]

\[ \Delta E = \frac{3}{4} 13.6 \text{eV} \quad \leftarrow \text{PHOTON RADIATED} \]

\[ \Delta E = -13.6 \text{eV}(\frac{1}{n_e^2})^2 - (-13.6 \text{eV}(\frac{1}{n_f^2})^2) \]

\[ \Delta E = 13.6 \text{eV} \left( \frac{1}{n_f^2} - \frac{1}{n_e^2} \right) \]

\[ \Delta E > 0 \quad \text{photon radiated} \]

\[ \Delta E < 0 \quad \text{absorbed} \]

\[ \Delta E = h\nu = \frac{hc}{\lambda} \quad \Rightarrow \quad \frac{1}{\lambda} = \frac{13.6 \text{eV}}{h\nu} \left( \frac{1}{n_f^2} - \frac{1}{n_e^2} \right) \]

\[ = \frac{13.6 \text{eV}}{12160 \text{eVnm}} \left( \frac{1}{n_f^2} - \frac{1}{n_e^2} \right) \]

\[ \frac{1}{\lambda} = 91.27 \text{nm} \left( \frac{1}{n_f^2} - \frac{1}{n_e^2} \right) \]

\[ R \]

\[ \text{Rydberg Constant} \]

\[ \text{Hydrogen} \]
BOHR MODEL
So we can make a schematic diagram

\[ \Delta E = E_{\text{INIT}} - E_{\text{FINAL}} \]

\[ \frac{1}{\lambda} = \frac{1}{91,76} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \]

Much of your homework for this week is to manipulate these equations.

Note model explains spectral lines.

Recall

\[ \frac{1}{\lambda} = \frac{1}{R_D} \left( \frac{1}{2^2} - \frac{1}{m^2} \right) \]

\[ \frac{1}{\lambda} = \frac{1}{91,127} \left( \frac{1}{2^2} - \frac{1}{3^2} \right) \]

\[ \text{Balmer series} \]

\[ m = 3, 4, 5 \ldots \]

\[ \frac{656.46 \text{ nm}}{656.46 \text{ nm}} = \text{Hydrogen \lambda line (RED line)} \]
WHAT ABOUT OTHER ATOMS?

DOES THIS WORK?

YES IF TWO PARTICLE SYSTEM

GENERAL TERM

\[ E_n = \frac{m_e e^4}{2(4\pi \varepsilon_0 h^2)^2} \frac{1}{n^2} \]

\[ E = \frac{m(Ze)^2 (Ze)^2}{2(4\pi \varepsilon_0 h^2)^2} \frac{1}{n^2} \]

So would work for isotopes of Hydrogen

1. hydrogen, deuterium, tritium
2. ionized atoms \((\text{He})^+\)
3. doubly ionized \((\text{Li})^{++}\)

Exotic atoms

one of your homework problems deals w/ this
Bohr model does not work for multi-electron atoms. Why?

$U_e$ is now a function of $r_i, r_3$

$r_3$ changes with time as the electrons move

Can't solve analytically

Could solve with computer, stepping from some initial configuration

However, Bohr model showed that quantization of angular momentum leads to a model that describes spectral lines of hydrogen.

On the right track

Also the expression for $E_n$ is the one that falls out of a quantum mechanical treatment using wave functions + Schrödinger Equation
One more point

NOT REALLY CORRECT TO SAY THAT THE ELECTRON ORBITS THE NUCLEUS

ACTUALLY THE ORBIT A CENTER OF MASS COM

So there is a small correction

\[ m_e \rightarrow \frac{m_e m_p}{m_e + m_p} \]

reduced mass

we will not include this in our discussion of the Bohr Model

However \rightarrow \text{Physics Majors + those of you who will work w/ spectroscopy should be aware of it}