Now we come to
ticks and tocks, sir.
Try to say this
Mr. Knox, sir. . . .
SPECIAL RELATIVITY

Length + time continued

From last time we defined two useful expressions:

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ \beta = \frac{v}{c} \]

Also derived:

\[ t = \gamma t' \]

"Moving clocks run slow as seen by observer at rest."

SEE L5 for derivation.

The effect is called "time dilation" and it is not limited to light and rockets.

→ Applies to any clock in an inertial reference frame.

(in the case of the rocket, the clock was the light pulse bouncing back and forth.)

Could be a pocket watch, if precise enough;

Could be an atomic clock;

Could be an unstable atomic particle

(see example measuring muons at different elevations.)
Time-dilation effect is insignificant at low speeds...

...but becomes overwhelming at speeds close to $c$.

<table>
<thead>
<tr>
<th>$\frac{V}{C}$ ($\beta$)</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>1.15</td>
</tr>
<tr>
<td>0.90</td>
<td>2.29</td>
</tr>
<tr>
<td>0.99</td>
<td>7.09</td>
</tr>
</tbody>
</table>
Manufacturer: Lockheed “Skunk Works”

6 March 1990
Los Angeles to Washington D.C.
Time: 1 hour 4 minutes 19.89 seconds
Average Speed: 2,144.83 mph
Distance Covered 2299.67 miles

Suppose the 1 hour 4 minutes 19.89 seconds was measured by a clock on the ground.

Would a clock on the plane have registered more time? less time?

By how much?
Example problem with SR-71

$\Delta T_E = 1\ h\ 4\ min\ 19.84\ s = 3860\ s$

$V = 2144.83\ \text{mi/hr}$

$\Delta T_E = \gamma \Delta T_P$ 

$\Delta T_P = \frac{\Delta T_E}{\gamma}$

$\Delta T_P = \Delta T_E \left(1 - \left(\frac{V}{c}\right)^2\right)^{\frac{1}{2}}$

Use Taylor Series Expansion

$(1 + x)^{\frac{1}{2}} \approx 1 + \frac{1}{2} x + H.O.\ Terms$

$x = -\frac{V^2}{c^2} \quad x < 1$

$\Delta T_P = \Delta T_E \left(1 - \frac{1}{2} \frac{V^2}{c^2}\right)$

$\Delta T_P - \Delta T_E = -\frac{\Delta T_E}{2} \frac{V^2}{c^2}$

or

$\Delta T_E - \Delta T_P = \frac{\Delta T_E}{2} \frac{V^2}{c^2}$

Fewer - signs to think about

$= 3860\ s \cdot \frac{1}{2} \left(\frac{2144.83\ \text{mi/hr}}{186,000\ \text{mi/s}} \cdot \frac{3600\ \text{s}}{\text{hr}}\right)^2$

$= 19.8 \times 10^{-9}\ s = 19.8\ \text{ns}$

⇒ Time seen on Earth is 19.8 ns longer than time measured on plane.
Time dilation leads to several paradoxes (puzzles).

One is called the **Twin Paradox**

Two twins **JACK** **JILL**

\[
\begin{align*}
\text{JILL GOES TO STAR W} & \quad \text{V = 0.5c} \\
\text{FAST ROCKET AND RETURNS} & \quad \text{4 light years}
\end{align*}
\]

**WHAT DOES THIS LOOK LIKE TO JACK?**

JILL is **gone** \( \frac{8 \text{ light years}}{0.5c} = 16 \text{ years} \)

So JACK aged 16 years.

But JILL is moving, her "clock" runs slow.

\[
\begin{align*}
T'_{\text{JILL}} &= T_{\text{JILL}}, \\
T'_{\text{JILL}} &= T_{\text{JACK}} \gamma \\
\gamma &= \frac{1}{\sqrt{1 - (0.5c)^2}} = 1.15 \\
16 \text{ yrs}/1.15 &= 13.9 \text{ yrs}
\end{align*}
\]

So JILL aged only 13.9 or about 2 years less than Jack.

\[\Rightarrow\text{SO WHAT IS THE PARADOX?} \]

**CONSIDER WHAT HAPPENS FROM JILL'S PERSPECTIVE, SHE REMAINS AT REST!**

**JACK HAS BEEN TRAVELING AT 0.5c RELATIVE TO HER!**

Shouldn't JACK be younger?
So apparent paradox is why is Jill younger + not Jack?

(discuss)

Simple explanation

Not symmetric. Jill made a trip, Jack did not.


---

The difference is that Jack stayed in inertial frame; Jill was in 2 different inertial frames.

So you have to work the problem that way. Jill did age less than Jack.
For SR-71 plane both observer + pilot are in inertial r. frames

Pilot earth moving past at $V$

$$V \Delta T_p = L_p$$

reca! $\Delta T_p < \Delta T_E$

so $L_p < L_E$

Ground

$$V \Delta T_G = L_G \quad (= L)$$

$$\frac{V \Delta T_p}{\Delta T_G} = \frac{L_P}{L_G}$$

$$\Delta T_G = \gamma \Delta T_p$$

$$\frac{V \Delta T_G}{V \Delta T_p} = \frac{L_G}{L_P} \implies \gamma = \frac{L_G}{L_P}$$

Pilot sees distance shortened by $\frac{1}{\gamma}$

more generally

$$L = \frac{1}{\gamma} L_0$$

$L_0$ length at rest

$L$ length as observed when $L_0$ moving at $\gamma$
so how much shorter is $L_p$ than $L_0$

\[ L_p = \frac{1}{\beta} L_0 \]

should be

$L_p = \sqrt{1-v^2/c^2} L_0$

(L. Wiencke 9/12/2012)

\[ = \left( 1 - \frac{1}{2} \frac{V^2}{c^2} \right) L_0 \quad \text{SEE P2 (same approximation)} \]

\[ L_p - L_0 = -\frac{1}{2} \frac{V^2}{c^2} L_0 = -\frac{1}{2} \left( \frac{2144.83 \text{ mi/hr}}{186,000 \text{ mi/hr}} \right)^2 \times 1.998 \times 10^6 \text{ m} \]

Plug in numbers and get

\[-20.5 \times 10^{-6} \text{ m}\]

or $L_p$ is 20.5 mm shorter than $L_0$

(this is the distance that would be covered in 19.8 ns traveling at 2145 mph)
“Bubble Chamber” Recording of tracks made by particles produced at a particle accelerator. Only charged particles are visible.
The bubble chamber picture of the first omega-minus particle detected.

SYNCHRONIZATION OF CLOCKS

Moving clocks run slow
Moving lengths appear shorter
Another effect is time synchronization

Simple example is how we view clock in back of class -
this depends on distance to the clock for observers

But there is another effect

\[ \text{Clock} \quad \leftarrow \frac{L}{2} \rightarrow \leftarrow \frac{L}{2} \rightarrow \text{Clock} \]

\[ \text{FLASH LIGHT} \]
\[ \text{START CLOCKS} \]
\[ \text{WHEN LIGHT HITS THEM} \]

Suppose they are not moving -
Observers will agree that clocks are synchronized

Suppose clocks are moving
Are the clocks still synchronized to observer at rest?
Three things are going on in this example

1. Train looks shorter \( L_{obs} = \frac{L}{\gamma} \)
2. Clocks look slow \( T_{obs} = \frac{T}{\gamma} \)
3. Train is moving

To observer on train \( T_{F} - T_{B} = 0 \) (synchronized)

Consider the front clock (F) what does observer see?

Light must travel from source to (F) \( \frac{L}{2} \) Light distance will be a bit more than \( \frac{L}{2} \) since the train moves some distance \( \Delta x \) during the time it takes the light to reach (F)

\[ t_{FC} + t_{FV} = \frac{L}{2} \cdot \frac{1}{\gamma} \]

To observer shortened because train is moving

For back clock (B)

\[ t_{BC} - t_{BV} = \frac{L}{2} \cdot \frac{1}{\gamma} \]

Now do some algebra.

We want to find \( t_{F} - t_{B} \)
\[ t_f (c+v) = \frac{1}{2} \frac{L}{\gamma} \]

\[ t_B (c-v) = \frac{1}{2} \frac{L}{\gamma} \]

\[ \Delta t_{\text{obs}} = t_f - t_B \]

\[ t_f = \frac{L}{2\delta} \frac{1}{c+v} \quad t_B = \frac{L}{2\delta} \frac{1}{c-v} \]

\[ t_f - t_B = \frac{L}{2\delta} \left( \frac{1}{c+v} - \frac{1}{c-v} \right) \]

\[ = \frac{L}{2\delta} \frac{c-v - (c+v)}{(c+v)(c-v)} \]

\[ = \frac{L}{2\delta} \frac{-2v}{c^2 - v^2} \]

\[ = -\frac{L}{2\delta} \frac{2v}{c^2} \frac{1}{\sqrt{1-v^2/c^2}} \]

\[ \delta = \sqrt{\frac{1}{1-v^2/c^2}} \]

\[ -\frac{L}{c^2} \frac{2v}{\gamma^2} \]

\[ \Delta t_{\text{obs}} = -\frac{\gamma LV}{c^2} = t_f - t_B \]

Note:

If \( v = 0 \), \( \Delta T = 0 \) so \( t_B \) started first.
THE (NO SO) GREAT TRAIN ROBBERY

THE TRAIN WAS 2000 M LONG - CARRYING GOLD (ON ONE CAR)

BUT VERY FAST

Robbers had taken PH300. Forgot most of it

\[ L = L_0 \gamma \]

\( \gamma \) was large fast train

So large train would fit in tunnel

Close doors

Capture gold!

WHAT ABOUT THE SITUATION SEEN FROM TRAIN?

TUNNEL MOVING FAST, LOOKS EVEN SHORTER, NO WAY WILL TRAIN FIT IN TUNNEL SO NO PROBLEM

QUESTION IS - WHAT HAPPENS?
Neutrino outruns speed of light, scientists say

If finding is confirmed, pillar of physics falls

By Frank Jordans
and Seth Borenstein
The Associated Press

GENEVA — A pillar of physics — that nothing can go faster than the speed of light — appears to have been smashed by an oddball subatomic particle that has apparently made a giant end run around Albert Einstein’s theories.

Scientists at the world’s largest physics lab said Thursday that they have clocked neutrinos traveling faster than light. That’s something that according to Einstein’s 1905 special theory of relativity — the famous “E=mc^2” equation — just doesn’t happen.

“The feeling that most people have is this can’t be right, this can’t be real,” said James Gillies, a spokesman for the European Organization for Nuclear Research.

SPEED » 11A

Fiery finish: Bus-size satellite to crash to Earth today. » 11A
What are the scientists saying?

"We tried to find all possible explanations for this," the report's author Antonio Ereditato of the Opera collaboration told BBC News on Thursday evening.

"We wanted to find a mistake - trivial mistakes, more complicated mistakes, or nasty effects - and we didn't.

"When you don't find anything, then you say 'well, now I'm forced to go out and ask the community to scrutinise this'."
Cern test ‘breaks speed of light’

0.0024 seconds 0.00000006 seconds 732 km

Time taken by neutrinos faster than the expected time distance travelled through rock

Cern, Geneva
Gran Sasso

Cern, Switzerland: A beam of neutrino particles is sent through rock towards Italy

Gran Sasso, Italy: Bricks with ultrasensitive covering at underground laboratory detect arrival
Update on the OPERA measurement of faster (?) than light neutrinos
Fig. 5: Schematic of the time of flight measurement.
Two clocks, separated by some distance, synchronized by a moving clock.

We did something similar in Special Relativity. What was it?

NOTE: This may or may not be an issue for the OPERA project. Depends on details of the clocks and GPS systems. It is quite possible this effect has already been taken into account. But it is still interesting to think about.
Observer at Rest

Observer Moving at V

Separation is 0
Everyone agrees
clocks are synchronized
(Correct for gammat term)
Observer Moving at $V$
Sees clocks different by $\frac{VL}{c^2}$

Observer at Rest

Distance L
GPS

common view

Satellite Moving at V
Used to synchronize clocks
Clocks now synchronized relative to satellite.

Observer
at Rest
Sees clocks
different by $\frac{V L}{c^2}$

Distance L
Observer at Rest sees clocks different by $\frac{VL}{c^2}$.

Satellite moving at $V$ is used to synchronize clocks.

Clocks are now synchronized relative to the satellite.

Plug in $V = 4000 \text{ m/s}$, $L = 730,000 \text{ m}$, $c = 3 \times 10^8 \text{ m/s}$, and you get $\sim 30 \text{ ns}$. 
NOTE: The clock synchronization $VL/c^2c$ may not be an issue for the OPERA project. Depends on details of the clocks and GPS systems. Which satellites are used and the ones used travel parallel to CERN/Gran Sasso line.

It is quite possible this effect has already been taken into account or cancels because many different satellites are used. But it is still interesting to think about.
BREAKING NEWS: Error Undoes Faster-Than-Light Neutrino Results

by Edwin Cartlidge on 22 February 2012, 1:45 PM | 222 Comments

It appears that the faster-than-light neutrino results, announced last September by the OPERA collaboration in Italy, was due to a mistake after all. A bad connection between a GPS unit and a computer may be to blame.

Physicists had detected neutrinos travelling from the CERN laboratory in Geneva to the Gran Sasso laboratory near L'Aquila that appeared to make the trip in about 60 nanoseconds less than light speed. Many other physicists suspected that the result was due to some kind of error, given that it seems at odds with Einstein's special theory of relativity, which says nothing can travel faster than the speed of light. That theory has been vindicated by many experiments over the decades.
Once Again, Physicists Debunk Faster-Than-Light Neutrinos

by Adrian Cho on 8 June 2012, 3:39 PM | 17 Comments

Enough already. Five different teams of physicists have now independently verified that elusive subatomic particles called neutrinos do not travel faster than light. New results, announced today in Japan, contradict those announced last September by a 170-member crew working with the OPERA particle detector in Italy’s subterranean Gran Sasso National Laboratory. The OPERA team made headlines after they suggested neutrinos traveled 0.002% faster than light, thus violating Einstein’s theory of special relativity. The OPERA results were debunked months ago, however. So instead of the nail in the coffin of faster-than-light neutrinos, the