Happy Labor Day!
The Special Theory of Relativity
Michaelson-Morley Experiment concluded:

- Speed of light same in all directions
- No Ether
- No Preferential Reference Frame
3 best known papers:
Explanation of Photoelectric Effect
Motion of Molecules (Brownian Motion)
Special Relativity
Albert Einstein (1879–1955)

- At the age of 16, Einstein began thinking about the form of Maxwell’s equations in moving inertial systems.

- Said to have been unaware of the null result of the Michelson-Morley experiment (he was 2 years old, when Michelson first reported a null result).

- In 1905, at the age of 26, he published his startling proposal about the principle of relativity, which he believed to be fundamental.
Einstein’s Postulate

1. The principle of relativity:
The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.

2. The constancy of the speed of light:
Observers in all inertial systems measure the same value for the speed of light in a vacuum.
1. **The principle of relativity:**
   The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.

2. **The constancy of the speed of light:**
   Observers in all inertial systems measure the same value for the speed of light in a vacuum.
$V'_x = 90 \text{ mi/hr}$

$U = 60 \text{ mi/hr}$
\[ V'_x = 90 \text{ mi/hr} \]

\[ U = 60 \text{ mi/hr} \]

\[ V_x = V'_x + U = 150 \text{ mi/hr} \]
V' \_x = 186,000 \text{ mi/hr} = 3 \times 10^8 \text{ m/s} = c

U = 60 \text{ mi/hr}
$V_x' = 186,000 \text{ mi/hr}$

$U = 60 \text{ mi/hr}$

186,060 mi/hr (Galileo)

186,000 mi/hr (Einstein)
$V'_x = 186,000$ mi/hr

$V = 286,000$ mi/hr (Galileo)

$186,000$ mi/hr (Einstein)
\( v = 0.6c \)

\( \gamma = 0.6c \)
\[ V'_x = 0.6 \, c \]

\[ U = 0.6c \]

1.2c (Galileo) (Crossed Out)

0.88c (Einstein)
$V'_x = 90 \text{ mi/hr}$

$U = 60 \text{ mi/hr}$

150.0000 mi/hr (Galileo)

150.0000 mi/hr* (Einstein)

* But will be different if enough decimal places are calculated
Time-dilation effect is insignificant at low speeds...

...but becomes overwhelming at speeds close to c.
Recall Michaelson Morley (MM) Expt

conclusions:

speed of light did not change as device was rotated or operated at different seasons

**NO ETHER**

**NO PREFERRED REF. FRAME**

(no ether, ether was to provide a preferential reference frame, so no preferred Ref. Frame either)

SEE MM applet demo on course website
Albert Einstein

Doctorate 1905  Swiss Patent Office 1902-1909

3 best known papers

- Photoelectric Effect
- Motion of Molecules (Brownian motion)
- Special Relativity
  (unaware of interest in MM expt.)

Which one won him Nobel prize?

Concerning Relativity he proposed 2 postulates
(we would call them laws)

1. Laws of Physics are the same in all inertial reference frames

2. Speed of Light in free space has the same value $c$ in all inertial reference frames

Postulate 1 motivated especially by Maxwell's equations that describe Electro Magnetic radiation.

$$ c = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} $$

in Maxwell's Equations

For (1) to hold for M's equations, (2) follows from (1)

$\Rightarrow$ Postulate (2) leads to counterintuitive transformations of length & time
postulate # 2 is counter intuitive

\[ x = x' + ut \]
\[ x' = x - ut = \frac{dx'}{dt} = \frac{dx}{dt} - u \Rightarrow v'_x = v_x - u \]
\[ y' = y \]
\[ z' = z \]
\[ t' = t \]

These are known as Galilean Transforms. And the work quite well for "every day" speeds and times.

Base balls thrown from trains, waves on a moving rope

But we also know from various experiments that postulate 2 is true (no experiment has found otherwise).

So what are the consequences of 2? i.e. keeping C fixed in all inertial reference frames?

What have to change?

\[ \text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} \]

⇒ keep speed of light same in all inertial reference frames, then D, T will have to change when you transform between inertial reference frames
And not just Distance + Time also

- simultaneity
- momentum
- Energy + mass
- addition of velocities
- maximum speed for a massive object

will have different definition upon transformation between inertial reference frames.

OK, BUT WE KNOW THAT GALILEAN XFORMS WORK WELL FOR TRAINS + BASEBALLS.

RIGHT. SO WHATEVER NEW TRANSFORMS ARE USED TO BE CONSISTENT WITH EINSTEIN #2, MUST ALSO BE CONSISTENT WITH GALILEO WHEN U IS LLC AT THE LEVEL OF EVERYDAY EXPERIENCE
so is it really true that speed of light is same in all inertial ref frames? suppose source is moving, is $v_{obs}$ still $c$? what tests have been made?

GPS system is one example

- Receiver
- EARTH
- 20,000 km
- $V_{orbit} \approx 4\text{ km/s}$

(NOT IN GEOSYNCHRONOUS ORBIT, MOVING RELATIVE TO SURFACE OF EARTH)

receiver measures light arrival time from satellites

accuracy $\sim 5\text{ m}$ $\sim 15\text{ ns}$ timing accuracy $\quad c = \frac{1\text{ ft/s}}{1\text{ km/3\mu s}}$

$T = \frac{D}{V}$

$T$ Time for signal to receiver from satellite

$D$ Distance

$V$ Velocity of signal

$\frac{dT}{dV} = -\frac{D}{V^2}$

$\frac{dT}{dV} = -\frac{1}{V}$

$dT = -\frac{dV}{V}$

Suppose that we assume signals travel at $c + V_{orbit}$ rather than $c$

What would be the effect?

$V = 300,000 \text{ km/s}$ $\Delta V = 4 \text{ km/s}$

$T = \frac{20,000 \text{ km}}{300,000 \text{ km/s}} \approx \frac{1}{15} \approx 60,000 \text{ \mu s}$

$dT = -60,000 \text{ \mu s} \frac{4 \text{ km/s}}{300,000 \text{ km/s}} = \frac{4}{15} \text{ \mu s} \approx 800 \text{ \mu s}$

$\Rightarrow 800 \text{ ft error}$

much larger than 5m accuracy!
so for GPS, if one used $v_{signal} = c + v_{orbit}$ the resulting change would be ~800 ft, which is much larger than the 3m error that are typical.

Earlier tests done in 1960's. Why?

First particle accelerators available

accelerate protons (very small mass $1.6 \times 10^{-27}$ kg)

charged $\Rightarrow$ use $E, B, \vec{E}$ to acc.

Particle called a pi zero meson

$\pi^0$ is unstable, decays to two photons (light)

but $\pi^0$ created at $V$ close to $c$

suppose $V = 0.75c$

$\pi^0 \rightarrow \gamma \ \gamma \ \gamma V_{\gamma} = c$

measure $V_{\gamma}$, do you see $V_{\gamma} \neq c$?

Ans NEVER
derivation of time dilation

Consider a strangely shaped rocket moving at \( v \) relative to an observer at rest. We place the rocket in the \( ' \) system.

There is a simple experiment on the rocket that measures light travel time (but better than we did in class).

**What does observer see?** i.e. what is \( \Delta t \)?

To observer the light has not traveled \( 2L \), but further because the rocket is moving.

So observer sees:

\[
\Delta t = \frac{2L}{c} = \frac{2 \sqrt{L^2 + (\Delta t v)^2}}{c}
\]

\[
\Delta t^2 = \frac{4L^2}{c^2} + \frac{2(\Delta t v)^2}{c^2} = \frac{4L^2 + (\Delta t v)^2}{c^2}
\]

**do some algebra!**
\[ \Delta t^2 c^2 = 4 L^2 + \Delta t^2 v^2 \]

\[ \Delta t^2 (c^2 - v^2) = 4 L^2 \]

\[ \Delta t^2 = \frac{4 L^2}{c^2 - v^2} \]

Divide top + bottom by \( c^2 \)

\[ \Delta t^2 = \frac{4 \frac{L^2}{c^2}}{1 - \frac{v^2}{c^2}} \]

\[ \Delta t = \frac{2 \frac{L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \] but recall that \( \Delta t' = \frac{2 \frac{L}{c}}{c} \)

\[ \Delta t = \Delta t' \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

try an example...

suppose \( \Delta t' = \mu s \) \( v = 0.9c \) really fast rocket

\[ \Delta t = \mu s \frac{1}{\sqrt{1 - (0.9c)^2}} = \mu s \frac{1}{\sqrt{1 - 0.81}} \]

\[ \Delta t = \mu s \frac{1}{\sqrt{0.19}} = 2.29 \mu s \]
Spaceship clock: Tick = 1 ms

Obs.: 1 Tick = 2.24 ms

Moving clocks appear to run slow

or

Moving clocks, as seen by observer at rest, run slow.

Where is this effect seen most commonly?

Particle physics experiments

Note: \( \Delta t = \Delta t' \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \)

Suppose \( v << c \) what happens?

Another S.R. quantity

\( B = \frac{v}{c} \)

For light \( B = 1 \)
A early (and famous) observation of time dilation effects was observed in a famous experiment performed in Colorado in the 1940s by B. Rossi.

It involved measuring a particle called the muon. Muons are produced when very high energy particles (cosmic rays) strike the atmosphere. They tend not to re-interact with material, except through ionization. They are unstable with a lifetime, $\tau$, of $2.2 \mu$s ($2.2 \times 10^{-6}$ s) for a muon at rest.

The experiment was to measure the rate of muons at mountain altitudes (Echo Lake) and in Denver. A simplified sketch is shown below.

Suppose counter 1 measures 19000 muons/hour.

How many would be measured by counter 2 in Denver?

A) Assuming Classical Relativity (Galileo)
B) Assuming Special Relativity (Einstein)

Assume muons travel straight down neglect effect of atmosphere.

Hints: $N(t) = N_0 e^{-t/\tau}$

$N(t)$ # muons vs time

$\tau$ lifetime of muon

t time

→ Work in observer rest frame

→ You will need to find height of Echo Lake + Height of Denver