Castle Bravo Hydrogen Bomb
March 1\textsuperscript{st} 1954 Bikini Atoll

Largest US Atomic Test
15 Megaton Yield
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(1 Megaton = energy released by a million tons of TNT = $4\times10^{15}$ Joules)

How much mass was converted to Energy?

A. 70g  B. 700g  C. 7 kG  D. 70 kG  E. 700 kG
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Mass equivalent
\[ m = \frac{E}{c^2} = \frac{15 \times 4 \times 10^{15}}{(3 \times 10^8)^2} = \frac{6 \times 10^{16}}{9 \times 10^{16}} = 670 \text{ g} \]

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When doing relativistic kinematics, use a unit called **Electron Volt (eV)**.

Particle with charge of 1 electron will be accelerated across electrostatic potential will gain energy.

1 electron volt = 1 volt x electron charge

\[
1 \text{ eV} = \frac{1 \text{ joule}}{1 \text{ coulomb}} \times 1.6 \times 10^{-19} \text{ coulomb}
\]

\[
1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}
\]

Also,

\[
6 \times 10^{18} \text{ eV} = 1 \text{ J}
\]

A convenient unit when dealing with particles that have been accelerated across voltages (as particles are in particle accelerators).

\[
1 \text{ KeV} = 10^3 \text{ eV} \quad \text{Kilo}
\]
\[
1 \text{ MeV} = 10^6 \text{ eV} \quad \text{Mega}
\]
\[
1 \text{ GeV} = 10^9 \text{ eV} \quad \text{Giga}
\]
\[
1 \text{ TeV} = 10^{12} \text{ eV} \quad \text{Tera}
\]
\[
1 \text{ PeV} = 10^{15} \text{ eV} \quad \text{Peta}
\]
\[
1 \text{ EeV} = 10^{18} \text{ eV} \quad \text{Exa}
\]
\[
1 \text{ ZeV} = 10^{21} \text{ eV} \quad \text{Zeta}
\]

\[
1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}
\]
\[
6 \times 10^{18} \text{ eV} = 1 \text{ J}
\]
1. PHYSICAL CONSTANTS

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol, equation</th>
<th>Value</th>
<th>Uncertainty (ppb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light in vacuum</td>
<td>$c$</td>
<td>299 792 458 m s$^{-1}$</td>
<td>exact*</td>
</tr>
<tr>
<td>Planck constant</td>
<td>$h$</td>
<td>6.626 068 96(33)×10$^{-34}$ J s</td>
<td>50</td>
</tr>
<tr>
<td>Planck constant, reduced</td>
<td>$h = h/2\pi$</td>
<td>1.054 571 628(53)×10$^{-34}$ J s</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 6.582 118 99(16)×10$^{-22}$ MeV s</td>
<td>25</td>
</tr>
<tr>
<td>electron charge magnitude</td>
<td>$e$</td>
<td>1.602 176 487(40)×10$^{-19}$ C = 4.803 204 27(12)×10$^{-10}$ esu</td>
<td>25, 25</td>
</tr>
<tr>
<td>conversion constant</td>
<td>$hc$</td>
<td>197.326 9631(49) MeV fm</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>$(hc)^2$</td>
<td>0.389 379 304(19) GeV$^2$ mbarn</td>
<td>50</td>
</tr>
<tr>
<td>electron mass</td>
<td>$m_e$</td>
<td>0.510 998 910(13) MeV$/^c^2$ = 9.109 382 15(45)×10$^{-31}$ kg</td>
<td>25, 50</td>
</tr>
<tr>
<td>proton mass</td>
<td>$m_p$</td>
<td>938.272 013(23) MeV$/^c^2$ = 1.672 621 637(83)×10$^{-27}$ kg</td>
<td>25, 50</td>
</tr>
<tr>
<td>deuterium mass</td>
<td>$m_d$</td>
<td>1.007 276 466 77(10) u = 1836.152 672 47(80) $m_e$</td>
<td>0.10, 0.43</td>
</tr>
<tr>
<td>unified atomic mass unit</td>
<td>(mass $^{12}$C atom)/12 = (1 g)/(($N_A$ mol)</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>permittivity of free space</td>
<td>$\varepsilon_0$</td>
<td>1 = $1/\mu_0c^2$</td>
<td>exact</td>
</tr>
<tr>
<td>permeability of free space</td>
<td>$\mu_0$</td>
<td>4$\pi$×10$^{-7}$ N A$^{-2}$ = 12.566 370 614 ... ×10$^{-7}$ N A$^{-2}$</td>
<td>exact</td>
</tr>
</tbody>
</table>

\[ m_e \, 0.510 \, 998 \, 910(13) \, \text{MeV} / c^2 = 9.109 \, 382 \, 15(45) \times 10^{-31} \, \text{kg} \]

\[ m_p \, 938.272 \, 013(23) \, \text{MeV} / c^2 = 1.672 \, 621 \, 637(83) \times 10^{-27} \, \text{kg} \]

E=$m_0\gamma c^2$ For particle at rest $\gamma=1$ so E=$m_0 c^2$

Or $m_0=E/c^2$ i.e. for a proton $m_0=938 \, \text{MeV} / c^2$
\[ m_e \ 0.510998910(13) \text{ MeV}/c^2 = 9.10938215(45) \times 10^{-31} \text{ kg} \]
\[ m_p \ 938.272013(23) \text{ MeV}/c^2 = 1.672621637(83) \times 10^{-27} \text{ kg} \]

\[ E = m_0 \gamma c^2 \quad \text{For particle at rest } \gamma = 1 \text{ so } E = m_0 c^2 \]

Or \[ m_0 = E/c^2 \quad \text{ie for a proton } m_0 = 938 \text{ MeV}/c^2 \]
700g of mass
Converted in much much much
Less than a second

200 g of mass
Converted per year
Windsor Ontario – 580 megawatt natural gas-fired power plant. What is the mass equivalent of the energy this plant generates per year?

580 megawatts = $6 \times 10^8$ J/s x 3 x $10^7$ s/yr = $1.8 \times 10^{16}$ J/yr

$E = mc^2$

$m = \frac{E}{c^2}$

$= \frac{1.8 \times 10^{16} \text{J}}{(3 \times 10^8 \text{m/s})^2}$

$= \frac{(1.8 \times 10^{16})}{9 \times 10^{16}} \text{kg}$

$= 200 \text{g}$

a. 200 g
b. 2 kg
c. 20 kg
d. 200 kg
Anti-matter The Energy source of the future??

Suppose there was an easy way to store anti-matter.

And suppose the reaction
\[(\text{matter} + \text{anti-matter}) \rightarrow \text{Energy}\]
could be controlled and harnessed.

Can we replace

With 100 grams of anti-matter??

Would anti-matter be a viable energy source??
Anti-matter  The Energy source of the future??

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Anti-matter The Energy source of the future??

Can we replace With 100 grams of anti-matter??

Would anti-matter be a viable energy source??

NO! (Sorry Scotty)

Antimatter: Energy Source of the Future and Always will Be.
Last time I introduced the modification for kinetic energy: \[ KE = m_0 c^2 (\gamma - 1) \]

One of your next homework assignments will be to show that for \( v \ll c \): \( m_0 c^2 (\gamma - 1) \rightarrow \frac{1}{2} m_0 v^2 \)

Hint: use first order Taylor series expansion for \( \gamma \).

We have rest mass energy \( E = m_0 c^2 \) object at rest \( \gamma = 1 \quad \beta = 0 \)

Suppose \( \gamma \neq 1 \quad \beta \neq 0 \) object moving

\[
E = m_0 c^2 + KE = m_0 c^2 + m_0 c^2 (\gamma - 1) \\
= m_0 c^2 + m_0 c^2 \gamma - m_0 c^2 \\
E = m_0 \gamma c^2 \quad \text{object moving}
\]

You can also show that \( E^2 = p^2 c^2 + m_0^2 c^4 \) relativistic expression for momentum.

Also note that \( \gamma = \frac{E}{m_0 c^2} \) if you are given \( E \) and \( m_0 \), you can find \( \gamma \).
\[ \text{KE} = m_0 (\gamma - 1) c^2 \]

where does it come from?

USE TWO STARTING EQUATIONS

1. \[ F = ma \]
2. \[ \text{KEgain} = \int F \, dx \quad (\text{PH 100}) \]

rewrite 1 for SR, substitute into 2, integrate

\[ F = ma = m \frac{dv}{dt} \]

does this work for this derivation?

no, need to use

more general

\[ = \frac{d}{dt} (m \, v) = \left( \frac{d}{dt} m \right) v + m \frac{dv}{dt} = m_0 \frac{d\gamma}{dt} + m_0 \gamma \frac{dv}{dt} \]

\[ = m \text{ in Newtonian Mechanics} \]

recall for SR \[ m \rightarrow m_0 \gamma \] \[ P = mv \rightarrow m_0 \gamma v \] for example

and \[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{\gamma v}{c^2}\right)^2}} \] so \[ m_0 \gamma \] has a connection to \( \gamma \) which

\[ \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \]

\[ \gamma v = \frac{1}{2} \left(\frac{-2v}{c^2}\right) \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \]

\[ \text{KE} = \int_0^x \left[ m_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \frac{dv}{dt} + \frac{v}{c^2} \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \right] dx \]

\[ = \int_0^x \left[ m_0 \left(1 - \frac{v^2}{c^2}\right) + \frac{v^*}{c^2} \right] \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \]

\[ = \int_0^x m_0 \frac{dv}{dt} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \quad \text{this can be solved} \]
\[ kE = m_0 c^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right)^v \]

Solution to this integral

\[ = m_0 c^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \]

\[ = m_0 c^2 (\delta - 1) \]
Anti Matter - great source of energy?

What are advantages + disadvantages?

- Small mass \( \leq \) antiparticle
  
  \[ 100 \text{ g} + 100 \text{ g} \]

  very efficient!

- Difficult to handle
  
  \[ m + \bar{m} \rightarrow \text{BOOM} \]

  WHERE TO GET IT?

How TO GET IT?

Typically in a particle accelerator

Simple example

\[ \text{INIT} \quad 0 \rightarrow 0 \]

\[ P \]

\[ E = \gamma m_p c^2 \]

\[ E_\text{INIT} = 2 \gamma m_p c^2 \]

\[ \text{FINAL} \]

\[ \bar{P} \]

\[ \text{violates something called baryon conservation} \]

\[ \text{Baryon Number} = +1 \text{ for proton} \]

\[ -1 \text{ for antiproton} \]

\[ \text{INIT} = 2 (2 \text{ protons}) \]

\[ \Rightarrow \text{Final} \]

\[ 3 + (-1) = 2 \]

\[ \bar{P} \]

\[ \text{all particles at rest} \]

3 protons

1 antiproton

\[ P + \bar{P} \rightarrow PPP\bar{P} \]

Suppose all particles created at rest

\[ E_f = 3E_p + 1E_{\bar{p}} \]

\[ = 3m_p c^2 + 1m_{\bar{p}} c^2 \]

\[ m_p = m_{\bar{p}} \]

\[ E_f = 4m_p c^2 \]
So

\[ E_{\text{INIT}} = E_{\text{FINAL}} \]

\[ 2 \sigma \text{mpc}^2 = 4 \text{mpc}^2 \]

\[ \gamma = 2 \]

So initial state

\[ E_{\text{RB}} = 4 \text{mpc}^2 \]

\[ = \frac{2 \text{mpc}^2}{\text{Rest mass energy}} + \frac{2 \text{mpc}^2}{\text{Kinetic energy}} \Rightarrow 4 \text{mpc}^2 \Rightarrow 2 \text{mpc}^2 + \left( \frac{2 \text{mpc}^2}{3 \text{p}, 1\bar{p}} \right) \]

Released by pp annihilation

So what's the problem?

\[ \frac{\text{mpc}^2}{\text{Kinetic energy of moving protons}} \]

is the same as energy out in pp annihilation

\[ \Rightarrow \text{no net gain in energy} \]

Need to "mine" antimatter

\text{Anti Matter - An energy source of the future!}

And always will be
Example Problem

\[ m_{\pi^+} = m_{\pi^-} = 139.6 \text{ MeV/c}^2 \]
\[ m_{\kappa_0} = 135.0 \text{ MeV/c}^2 \]
\[ m_{\pi^0} = 497.6 \text{ MeV/c}^2 \]

\[ \kappa_0 \text{ meson at rest decays to } \pi^+ \pi^- \text{ mesons} \]

\[ \text{INIT} \quad \kappa_0 \quad \uparrow \pi^+ \quad \text{WHAT IS } K \pi^+, \pi^- \quad \text{FINAL} \quad \downarrow \pi^- \quad P_{\pi^+ \pi^-} \quad \text{momentum} \]

conservation of energy

\[ E_{\text{INIT}} = M_{\kappa_0} c^2 = E_{\text{FINAL}} = m_{\pi^+} c^2 + m_{\pi^-} c^2 + \gamma_{\pi^+} c^2 + \gamma_{\pi^-} c^2 \]

conservation of momentum

\[ P_{\text{INIT}} = 0 = P_{\text{FINAL}} = m_{\pi^-} \gamma_{\pi^-} + m_{\pi^+} \gamma_{\pi^+} \quad \Rightarrow \quad \gamma_{\pi^-} = -\gamma_{\pi^+} \]

\[ m_{\pi^-} = m_{\pi^+} \]

\[ \gamma_{\pi^-} = -\gamma_{\pi^+} \quad \Rightarrow \quad V_{\pi^-} = -V_{\pi^+} \]

so velocity is equal and opposite

\[ M_{\kappa_0} c^2 = m_{\pi^+} \gamma_{\pi^+} c^2 + m_{\pi^-} \gamma_{\pi^-} c^2 \]

\[ M_{\kappa_0} \gamma_{\pi^+} = 2 m_{\pi^+} \gamma_{\pi^+} \]

\[ \frac{M_{\kappa_0}}{2m_{\pi^+}} = \gamma_{\pi^+} \\
E_{\pi} = m_{\pi^+} \gamma_{\pi^+} c^2 = \frac{m_{\pi^+} M_{\kappa_0} c^2}{2m_{\pi^+}} \]

\[ = \frac{1}{2} M_{\kappa_0} \gamma_{\pi^+} = \frac{497.6 \text{ MeV}}{2 c^2} \]

\[ \Rightarrow \quad E = P c^2 + m_{\pi^+} c^2 \quad \text{to get } P \]
NOTE ON ENERGY AND MOMENTUM

FOR MASS = 0 PARTICLES (PHOTON)

\[ P = m_0 \gamma v \] is not well defined

for a photon. \( m_0 \rightarrow 0 \) \( \gamma \rightarrow \infty \)

INSTEAD USE

\[ E^2 = m_0^2 c^4 + P^2 c^2 \]

for \( m_0 = 0 \)

This becomes \( E = PC \) or \( P = E/C \)

for mass = 0 particles
Example Problem

$\pi^0$ meson at rest decays to two photons.

The mass of a $\pi^0$ is 135.0 MeV/c$^2$.

What is the momentum of photon #1? #2?

---

**INITIAL STATE**

$\pi^0 \to$

$V = 0$

$P = 0$

(at rest)

**FINAL STATE**

$\uparrow$ Photon #1

$\downarrow$ Photon #2

By conservation of momentum

$\vec{P}_{PH1} + \vec{P}_{PH2} = 0$

$\Rightarrow \vec{P}_{PH1} = -\vec{P}_{PH2}$

or $|P_{PH1}| = |P_{PH2}|$ magnitudes are equal

$\Rightarrow E_{PH1} = E_{PH2}$

**CONS OF ENERGY**

INITIAL FINAL

$E_{\pi^0} = 2E_{PH}$

$\frac{m_{\pi^0}c^2}{2} = E_{PH}$

$\Rightarrow E_{PH} = \frac{m_{\pi^0}c^2}{2}$

$\Rightarrow P_{PH} = \frac{1}{2} m_{\pi^0} c = \frac{(135.0 \text{ MeV}/c^2)c}{2} = 67.5 \text{ MeV}/c$

$P_{PH1}, P_{PH2}$