Bohr Model of the Atom

Niels Bohr
1885-1962  Denmark, US
1922 Nobel Prize in Physics
Niels Bohr’s postulates

- Existence of “stationary” states for the orbiting electrons

- Transitions between stationary states:
  \[ E = E_1 - E_2 = h\nu \]

- Classical laws of physics do not apply to transitions between stationary states

- The electron can only exist in orbits for which its angular momentum is given by:
  \[ L = | \hat{r} \times \hat{p} | = mv\tau = nh/2\pi \text{ with } n=1,2,3... \]
Spectral Lines of Hydrogen

By 1913, some of these lines were not observed. Still they were predicted by Bohr’s model.
• If an electron in a hydrogen atom makes a transition from some state to a lower state do the following increase or decrease
  – Kinetic energy
  – Potential energy
  – Orbital angular momentum
• If an electron in a hydrogen atom makes a transition from some state to a lower state do the following increase or decrease
  – Kinetic energy
    • **Increase**  KE is proportional to $1/r$   \( r \) is smaller
  – Potential energy
    • **decrease**    becomes more negative
  – Orbital angular momentum
    • **decrease**  \( L=nh/2\pi \)
What energy is required to ionize a hydrogen atom with an electron in the first excited state? (remove the electron from the atom?)
What energy is required to ionize a hydrogen atom with an electron in the first excited state? (remove the electron from the atom?)

$E_{\text{init}}$ is the n=2 state

$E_{\text{final}}$ is the n→$\infty$

$E_2 - E_{\text{inf}} = -13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{\infty^2} \right) = 3.4 \text{ eV}$
Planetary Model

- Force Applied to the electron:

\[ \vec{F}_e = \frac{-1}{4\pi \varepsilon_0} \frac{e^2}{r^2} \hat{e}_r = \frac{mv^2}{r} \]

- One deduces: \( v = \frac{e}{\sqrt{4\pi \varepsilon_0 mr}} \)

- Total energy with \( K = (1/2)mv^2 \):

\[ E = K + V = \frac{e^2}{8\pi \varepsilon_0 r} - \frac{e^2}{4\pi \varepsilon_0 r} = \frac{-e^2}{8\pi \varepsilon_0 r} \]
Failure of the classical (planetary) atomic model

- Atom (neutral) = nucleus (+q) + q electrons

- Assuming the Hydrogen atom:
  - The electron is attracted by the nucleus
  - Even in circular motion around the nucleus, the electron loses energy:
    - Radial acceleration: \( a_r = \frac{v^2}{R} \)
    - Classical e.m. theory: an accelerating charge continuously radiates energy, \( r \) decreases...

The electron would eventually crash into the nucleus
Failure of the planetary model

- Doomed because the electron radiates energy, while orbiting around the nucleus

- But:
  - There is some “truth” to it, since Rutherford was successful in describing the scattering experiment

- In 1913, Niels Bohr 1885-1962 postulates that the electrons may be in stable (non-radiating) circular orbits, called stationary orbits
Bohr Radius

• From Bohr’s postulate: \( v = n\hbar/mr \)

• Since: \( v = e / (\sqrt{4\pi \varepsilon_0 mr}) \) [Planetary Model]

• One can deduce the diameter of the hydrogen atom for stationary states:

\[
    r_n = \frac{4\pi \varepsilon_0 n^2 \hbar^2}{me^2} = n^2 a_0 \quad \text{with } a_0, \text{ the Bohr radius}
\]
Energy Levels of the Hydrogen atom

- Energy of the stationary states:

\[ E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv -\frac{E_0}{n^2} \]

with \( E_0 = -13.6 \text{ eV} \)

Emission of light: \( h\nu = E_{n_u} - E_{n_l} \)
Using: \( \frac{1}{\lambda} = \nu/c \),

\[ \frac{1}{\lambda} = \frac{E_u - E_\ell}{hc} = R_\infty \left( \frac{1}{n_\ell^2} - \frac{1}{n_u^2} \right) \]

\( R_\infty \), the Rydberg constant for Hydrogen
Spectral Lines of Hydrogen

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Quantum to Classical Mechanics?

- The Correspondance Principle (Bohr):

In the limits where classical and quantum theories should agree, the quantum theory must reduce to the classical result.
From Rutherford scattering experiments it was determined experimentally that
atoms mostly take up positive, small nucleus at center.

First let's consider another $\frac{1}{r^2}$ force. What?

Gravity

$$F = \frac{G m M}{r^2}$$

Imagine dropping or throwing a ball from a tall crane.

Next consider a crane and a very fast "throw".

Keeps falling around the earth. Goes into orbit.

accelerating? yes
$$\frac{dv}{dt} \neq 0$$
but direction is changing

so with an attractive $\frac{1}{r^2}$ force, you can have massive objects in orbit.
So this has similarities to an atom

consider two charged objects (opposite)

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} = -\frac{Ze^2}{4\pi \varepsilon_0} \frac{1}{r^2} \]

\[ F = ma \]
\[ F = m \frac{dv}{dt} \]

From this you can derive
\[ \frac{dv}{dt} = -\frac{v^2}{r} \]

(not derived here) acting in -direction

\[ F = m\left(\frac{-v^2}{r}\right) \]

force and \( \frac{dv}{dt} \) act in same direction

note \( e^- \) is slow enough that we don't need special relativity

\[ E = KE_0 + U \]
\[ = \frac{1}{2} m v^2 + U = \frac{1}{2} \frac{Ze^2}{4\pi \varepsilon_0} \frac{1}{r} + U \]

\[ KE = \frac{Ze^2}{8\pi \varepsilon_0 r} \]
So now we need $U$ (potential energy)
for a $\frac{1}{r^2}$ force.

Matters because total energy is $U + KE$.

In Quantum Mechanics we will encounter
$U$ in the Schrödinger Equation.

Different systems

- Simple Harmonic Oscillator (SHO)
- Hydrogen Atom
- Etc

have different expressions for $U$. 
**Review of Potential Energy and Charges**

**Same Sign Charges**

- **Repulsive Force**
  - Bring charges closer
  - Requires energy to do this
  - Potential energy increases
  - Must do work (add energy) to bring charges together

**Opposite Sign Charges**

- **Attractive Force**
  - When \( r \) gets smaller, potential energy decreases
  - Must do work (add energy) to pull them apart

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**Potential Energy Equation**

\[
U = \frac{q'q}{4\pi\varepsilon_0} \frac{1}{r}
\]

For hydrogen atom, \( q' = +Ze \), \( q = -e \)

\[
U = -\frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r}
\]

\[
U(r) = -\int_{\infty}^{r} \frac{Ze^2}{4\pi\varepsilon_0 r^3} \, dr'
\]

\[
= \left[ \frac{-Ze^2}{4\pi\varepsilon_0} \frac{1}{r'} \right]_{r'=\infty}^{r'}
\]

\[
= \left( \frac{-Ze^2}{4\pi\varepsilon_0} \right) \frac{1}{r} - \left( \frac{-Ze^2}{4\pi\varepsilon_0} \right) \frac{1}{\infty}
\]

\[
= \left( \frac{-Ze^2}{4\pi\varepsilon_0} \right) \frac{1}{r}
\]
So assemble all the parts

\[ E = \frac{1}{2} E + U \]
\[ = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{r} - \frac{e^2}{4\pi\varepsilon_0} \frac{1}{r} \]
\[ E = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{r} - \frac{2e^2}{8\pi\varepsilon_0} \frac{1}{r} = -\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r} = E \]

Oh? actually we have a problem.
Think about what accelerating charges do

wire \quad \text{or} \quad \text{x-rays (unbound system)}

accelerated charge RADIATES ENERGY!

\[ e^- \text{ in atom would radiate energy (classically)} \]

ATOM WILL COLLAPSE IN \(10^{10}\) sec!

Q: Why doesn't our solar system collapse?
both forces are \(\frac{1}{r^2}\)

\[ F_{\text{EM}} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2} \quad F_{\text{GRAV}} = G \frac{Mm}{r^2} \]

accelerated mass will radiate - gravity waves!

ANS: Gravity much weaker \( E-M \text{ Force } \sim 10^{36} \times \text{Stronger!} \)
Energy loss is tiny \( \text{trillions of years (radiated)} \) to change orbits
So since matter does not collapse, we need to make a change to this simple model.

Recall: Planck invoked quantization of energy to evade a different problem of radiated energy (UV catastrophe).

Niels Bohr invoked quantization of angular momentum.

\[ \vec{L} = \vec{r} \times \vec{p} \quad \vec{L} \text{ is angular momentum} \]

\[ P = mv \]

\[ L = rv = \frac{nh}{2\pi} \quad \text{or} \quad h = \frac{L}{2\pi} \]

\[ L = nh \quad n = 1, 2, 3, \ldots \]

Quantization of angular momentum

So what does this imply about \( E, r \)? Under this model \( E, r \) will also be quantized.

Start with \( \hbar E = \frac{1}{2} mv^2 = \frac{1}{8\pi\varepsilon_0} \frac{1}{r^2} \) (case where \( Z = 1 \))

Hydrogen-like atom

Use quantization of angular momentum and \( r (n, Z_0, m_e, \hbar, e) \)

\[ \frac{1}{2} mv^2 = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{r} \]

\[ r \nu = n\hbar \quad \nu = \frac{n\hbar}{mr} \]

\[ \frac{1}{2} m \left( \frac{n\hbar}{mr} \right)^2 = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{r} \]

Solve for \( r \)

\[ \frac{n^2 \hbar^2}{2mr^2} = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{r} \]

\[ \frac{n^2 \hbar^2 4\pi\varepsilon_0}{me^2} = r = n^2 a_0 \]

\[ a_0 = 0.0529 \text{ nm} \]

\[ r = a_0, 4a_0, 9a_0, 16a_0, 25a_0, \ldots \]

\[ n = 1, 2, 3, 4 \]
Example in a transition from $n_f = 3$ to $n_i = 2$

Is a photon radiated or absorbed?

What is the wavelength of photon?

$n_f = 3$, $n_i = 2$

$$h \nu = 13.6 \text{eV} \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\nu = \frac{c}{\lambda}$$

$$h \nu = 13.6 \text{eV} \left( \frac{1}{4} - \frac{1}{9} \right)$$

$$\frac{hc}{\lambda} = 13.6 \text{eV} \left( \frac{1}{4} - \frac{1}{9} \right)$$

$$\frac{1}{\lambda} = \frac{13.6 \text{eV}}{hc} \left( \frac{1}{4} - \frac{1}{9} \right)$$

$$\frac{1}{\lambda} = \frac{13.6 \text{eV}}{1240 \text{eVnm}} \left( \frac{1}{4} - \frac{1}{9} \right)$$

$$\lambda = 1.523 \times 10^{-3} \text{nm}^{-1} \Rightarrow \lambda = 656 \text{nm}$$

Red line in Hydrogen spectrum

more accurately

$$\lambda = \frac{64 \pi^2 \varepsilon_0^2 \hbar c}{m_e^4} \left( \frac{n_i^2 n_f^2}{n_i^2 - n_f^2} \right)$$

$$R_d = 1.097 \times 10^7 \text{nm}^{-1}$$