PHGN 300 Problem Set 2

1) A. Plot the data from the speed of light class activity. (show your plot). Since the apparatus did not work well the day we did this activity, you may wish to do this analysis using data collected during a previous year. This data is: (1.2m, 84ns) (16.5 m 142ns) (9.09m, 102ns) (6.8m 102 ns) (8.8m 126ns) where the distances and times are for the round trip.
B. Fit a straight line to the data and determine the speed of light using this data.
C. Extra Credit: Write a simple computer program to determine the bootstrap error in your measurement of the speed of light.
D. Extra Credit: Determine the speed of light from your measurements using the Michaelson interferometer in class.
E. Extra Credit: Determine the speed of light using a microwave oven and a bar of chocolate or plate of marshmallows. (use google to figure out how to do this) Include a picture that you take of the chocolate or marshmallows.

2) Determine the ratio $\beta = (v/c)$ for the following
   1. A car traveling at 110 km/hr
   2. A commercial jet traveling at 290 m/s
   3. A supersonic jet traveling at Mach 2.3 (Mach number = $v/v_{sound}$)
   4. The space shuttle traveling at 27,000 km/h
   5. An electron traveling 40 cm in 2 ns
   6. A proton traveling across the nucleus (10^{-14} m) in 0.35 x 10^{-22} s.

3) An astronaut must journey to a star that is 200 light-years from earth. What speed will be necessary if the astronaut wishes to age only 10 years during the trip?

4) A hanger for housing space ships is 100 m long. A space ship fits into the hanger, briefly, as it passes through at 0.99c. Will it fit if it stops? How long is it the space ship (at rest relative to the hanger)?

5) You have been hired to design and build a Michaelson-Morley apparatus. You will use a HeNe laser for the light source and the length of the arms (L) will be 100 meters.
   A. What is the wavelength of this type of laser?
   B. You have found a detector that can resolve a shift in the interference pattern as small as 0.005 fringes (wavelengths). This means that when the device is rotated, and should the interference pattern shifted by just $5/1000^\text{th}$ of a fringe, the detector could see it. Using the class notes or text, obtain an equation for the ether velocity $V_E$, as a function of the detector arm length L, laser wavelength $\lambda$, and speed of light, c.
   C. Using your equation calculate the smallest $V_E$ your device can measure. Express you answer in km/second.

6) Problem on the attached page.
A early (and famous) observation of time dilation effects was observed in a famous experiment performed in Colorado in the 1940's by B. Rossi.

It involved measuring a particle called the muon. Muons are produced when very high energy particles (cosmic rays) strike the atmosphere. They tend not to re-interact with material, except through ionization. They are unstable with a lifetime, $\tau$, of $2.2 \mu s$ ($2.2 \times 10^{-6}$ s) for a muon at rest.

The experiment was to measure the rate of muons at mountain altitudes (Echo Lake) and in Denver. A simplified sketch is shown below.

Assume muons travel straight down, neglect effect of atmosphere.

**Hint:** $N(t) = No e^{-t/\tau}$

$N(t)$ #muons vs time

$\tau$ lifetime of muon

$t$ time

$\Delta$ Height

Identical muon counters

→ work in observer rest frame

→ You will need to find height of Echo Lake + Height of Denver

**Suppose counter 1 measures 19000 muons/hour**

How many would be measured by counter 2 in Denver?

A) Assuming Classical Relativity (Galileo)

B) Assuming Special Relativity (Einstein)
2) Determine the ratio $\beta (v/c)$ for the following
   1. A car traveling at 110 km/hr
      $\beta = \frac{v}{c} = \frac{110000 \text{ m/hr}}{1 \text{ hr}/3600 \text{ s}} \cdot \frac{3.0 \times 10^8 \text{ m/s}}{} = 1.01852 \times 10^{-7}$

   2. A commercial jet traveling at 290 m/s
      $\beta = \frac{v}{c} = \frac{290 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = 9.67 \times 10^{-7}$

   3. A supersonic jet traveling at Mach 2.3
      (Mach number $= \frac{v}{v_{\text{sound}}}$)
      $\beta = \frac{v}{c} = \frac{2.3 \times 340 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = 2.607 \times 10^{-6}$

   4. The space shuttle traveling at 27,000 km/h
      $\beta = \frac{v}{c} = \frac{2.7 \times 10^7 \text{ m/hr}}{1 \text{ hr}/3600 \text{ s}} \cdot \frac{3.0 \times 10^8 \text{ m/s}}{} = 2.5 \times 10^{-5}$

   5. An electron traveling 40 cm in 2 ns
      $v = \frac{dx}{dt} = 0.4 \text{ m/} 2 \times 10^{-9} \text{ s} = 0.2 \times 10^8$
      $\beta = \frac{v}{c} = \frac{1.25 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = 0.667$

   6. A proton traveling across the nucleus (10^{-14} \text{ m})
      in 0.35 x 10^{-22} \text{ s.}
      $v = \frac{dx}{dt} = 10^{-14} \text{ m/} 3.5 \times 10^{-23} \text{ s} = 2.86 \times 10^8$
      $\beta = \frac{v}{c} = \frac{2.86 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = 0.952$
3) \( v = \gamma, \)
\( x = 200 \text{ light years} \)
\( t' = 10 \text{ years} \)
\( v = \frac{x'}{t'} \quad x' = x_0 \sqrt{1 - \frac{v^2}{c^2}} \)

\[
v = \frac{200 \text{ light years}}{10 \text{ years}} \sqrt{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad v = 20 \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[
v^2 = 400 c^2 (1 - \frac{v^2}{c^2}) = 400 c^2 - 400 v^2 \]

\[401 v^2 = 400 c^2 \]

\[v = \sqrt{\frac{400}{401}} c \approx 0.99815c \]
3) The space station circles the earth at an altitude of 350km and a speed of 27,000km/hr. Suppose a clock is placed on the space and synchronized with a clock on earth. What will be the difference, including the sign, between the two clocks after 10 years? (Assuming that both clocks are in inertial reference frames.)

Assume that $T_1$ is time on the space station. And $t = 10$ years. Then on the earth the clock go through the time $T_2$. $T_2 = 10\text{years}/(1-v^2/c^2)^{1/2} = 10.00000003125\text{years}$. Or you can assume that $T_1$ is the time on the earth and $T_1 = 10\text{years}$. Then the time on the space station should be $T_2 = 10\text{years}*(1-v^2/c^2)^{1/2} = 9.999999969\text{years}$. Thus the difference would be $T_2 - T_1 = 0.00000003125\text{years} = 0.09855\text{seconds}$. The clock on the space is slower than that on the earth.

4) A hanger for housing space ships is 200 m long. A space ship fits into the hanger as it passes through at 0.99c. Will it fit if it stops? How long is it the space ship?

The length of the ship at rest is $L$.

So $L = 200\text{m}/(1-v^2/c^2)^{1/2} = 1417.76\text{m}$
\[ T_1 = \frac{L}{C+Ve} + \frac{L}{C-Ve} \]

\[ T_2 = 2L \left( \frac{C}{Ve^2-Ve^2} \right) \]

\[ T_1 = \frac{L(C-Ve) + L(C+Ve)}{C^2-Ve^2} \]

\[ = \frac{2LC}{C^2-Ve^2} \]

\[ T_2 = 2LC \left( \frac{1}{\sqrt{C^2-Ve^2}} \right) \cdot \frac{1}{C} = 2L \left( \frac{1}{\sqrt{C^2-Ve^2}} \right) \]

Rewriting to put in terms of \( \frac{Ve}{C} \):

\[ T_1 = 2L \left( \frac{1}{C} \right) \left( \frac{1}{1+\left(\frac{Ve}{C}\right)^2} \right) \]

\[ T_2 = 2L \left( \frac{1}{\sqrt{1-\left(\frac{Ve}{C}\right)^2}} \right) \]

\[ T_1 - T_2 = 2L \left( \frac{1}{1-\left(\frac{Ve}{C}\right)^2} - \frac{1}{\sqrt{1-\left(\frac{Ve}{C}\right)^2}} \right) \]

\[ \text{if } \frac{Ve}{C} \ll 1 \]

\[ \frac{Ve}{C} \approx 1 - \frac{1}{2} \left(\frac{Ve}{C}\right)^2 \]
\[ T_i - T_2 = \frac{2L}{c} \left( \frac{1}{1-\beta^2} - \frac{1}{\sqrt{1-\beta^2}} \right) \]

\[ \Delta t = \frac{2L}{c} \left( \frac{1 - \sqrt{1 - \beta^2}}{1 - \beta^2} \right) \quad \text{let} \quad V \ll c \]

\[ \Delta t = \frac{2L}{c} \left( \frac{1 - (1 - \frac{1}{2} \beta^2)}{1 - \beta^2} \right) = \frac{2L}{c} \left( \frac{\frac{\beta^2}{2}}{1 - \beta^2} \right) \]

\[ \text{let} \quad 1 - \beta^2 = 1 \]

\[ \Delta t = \frac{L}{c} \left( \frac{\beta^2}{c^2} \right) \]

if \( \Delta t = \frac{5}{1000} \text{ fringes} \cdot \frac{\Delta}{c} \quad \Rightarrow \quad \Delta t \cdot c = \frac{5}{1000} (\lambda) \]

\[ \frac{\Delta t \cdot c}{L} = \frac{V^2}{c^2} \]

\[ \frac{5}{1000} \left( \frac{\Delta}{L} \right) = \frac{V^2}{c^2} \quad \Rightarrow \quad V = c \sqrt{\frac{\Delta}{L} \left( \frac{5}{1000} \right)} \]

\[ V_{\text{minimum}} = c \sqrt{\frac{\lambda}{L}} \quad \text{(resolution in fringes)} \]

for \( \lambda = 633 \text{ nm} = 633 \times 10^{-9} \text{ m} \)

\( L = 100 \text{ m} \)

\[ V = c \sqrt{6.33 \times 10^{-9} \left( \frac{5}{1000} \right)} = c \sqrt{3.16 \times 10^{-11}} \]

\[ V = 5.6 \times 10^{-6} \text{ c} \approx 1.6 \text{ km/s} \]
6) Finding $\Delta t$ in the muon event

a) Classically, $\Delta t^e = \Delta t = \frac{\Delta x}{v}$

- Altitude (echo lake) = 3230 m
- Altitude (denver) = 1609 m

$\Delta x = 1621 m = \Delta X$

$\Delta t = \frac{\Delta X}{0.98c} = 5.51 \times 10^{-6} s$

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

$N_0 = 10,000$

$\tau = 2.2 \times 10^{-6} s$

$N = 820$ muons/hr

b) Special relativity

$$\Delta t_{\text{earth}} = \Delta t_{\text{muon}} \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) \Rightarrow \frac{\Delta t_{\text{earth}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \Delta t_{\text{muon}}$$

$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 5$

$\Rightarrow \Delta t_{\text{earth}} / 5 = \Delta t_{\text{muon}}$

$\Rightarrow \Delta t_{\text{muon}} = 1.1 \times 10^{-6} s$

$N(t) = N_0 e^{-\frac{t}{\tau}} \Rightarrow N_0 e^{-\frac{t}{\tau}}$

$N = 6065$ muons/hr