Homework Problem Set #3 PHGN 300

1) This problem refers to the WileE Coyote example from class with the four \( B = 0.5 \) rockets. The slides and lecture notes are available on the course website.

There are several ways to calculate WileE's speed relative to the ground. Do NOT use a calculator for this problem.

- a) sequential calculation: Determine his speed as seen from rocket 2. Then using this value determine his speed as seen from rocket 1, and then finally from the ground.
- b) split calculation: determine his speed as seen from rocket 2 and the speed of the ground as seen from rocket 2. Then combine these speeds to determine his speed relative to the ground.
- c) are the results from a) and b) the same? Explain.

2) At what value of \( \beta \) does the first order Taylor series expansion (as used in class) for \( \gamma \) induce an error of 1% relative to using the actual definition? At what \( \beta \) does the error become 10%? Hint: You may find it easier to solve this numerically using Mathematica or a calculator.

3) What is the speed of an electron with kinetic energy of 3.4 KeV? 1 MeV? 10 MeV?

4) What speed must an object have so that its kinetic energy is equal to its rest mass?

5) The OPERA experiment found evidence, later traced to a loose fiber optic cable, that neutrinos that traveled from the CERN accelerator 450 km away arrived 60 ns ahead of the time that light would have taken. In Feb 23 1987 a supernova explosion (SN 1987A) was observed in the large Magellanic Cloud. About the same time a burst of neutrinos were observed from SN 1987A. If the OPERA result had been true, how much earlier would neutrinos have arrived from the supernova SN 1987A before the optical observations? Show your work and the value and reference you use to find the distance to SN 1987A.

Extra credit

Refer to (5) For SN 1987A, assume the neutrino kinetic energy was 10 MeV and neutrinos and the light were created at the same time. If the neutrinos arrived 1 hour after the optical observation what would be the mass of the neutrino?
Wile E Coyote Problem Solution

1) Draw a diagram to help you

\[ \begin{align*}
V_{43} &= 0.5c \\
V_{32} &= 0.5c \\
V_{21} &= 0.5c \\
V_{10} &= 0.5c \\
\text{EARTH (ground)}
\end{align*} \]

For example, \( V_{21} \) means speed of rocket 2 as seen from rocket 1.

Q) What is \( V_{46} \) (speed of wile E as seen from ground)?

A) Calculate \( V_{42} \) then \( V_{41} \), then \( V_{46} \)

\[ V_{42} = \frac{V_{32} + V_{43}}{1 + \frac{V_{32} \cdot V_{43}}{c^2}} = \frac{0.5c + 0.5c}{1 + \frac{0.5c \cdot 0.5c}{c^2}} = \frac{c}{\frac{5}{4}} \cdot \frac{4}{\frac{5}{4}} = \frac{4}{5} c \]

\[ V_{41} = \frac{V_{42} + V_{21}}{1 + \frac{V_{42} \cdot V_{21}}{c^2}} = \frac{0.5c + \frac{4}{5} c}{1 + \frac{0.5c \cdot \frac{4}{5} c}{c^2}} = \frac{\frac{9}{10} c + \frac{4}{10} c}{1 + \frac{4}{10} c} = \frac{13}{14} c \cdot 0.928c \]

\[ V_{46} = \frac{V_{46} + V_{41}}{1 + \frac{V_{41} \cdot V_{46}}{c^2}} = \frac{\frac{4}{5} c + \frac{13}{14} c}{1 + \frac{\frac{4}{5} c \cdot \frac{13}{14} c}{c^2}} = \frac{\frac{40}{41} c}{\frac{28}{28} + \frac{13}{28}} = 0.975c \]

B) Calculate \( V_{42} \), \( V_{46} \) then \( V_{46} \)

From A \( V_{42} = \frac{4}{5} c \), by symmetry \( V_{26} = V_{42} \) same

So \( V_{46} = \frac{\frac{4}{5} c + \frac{4}{5} c}{1 + \frac{4}{5} \cdot \frac{4}{5}} = \frac{5.4}{5.5} \cdot \frac{c + 5.4}{5.5} \cdot \frac{(20+20)c}{(25+16)c} = \frac{40}{41} c \)
Problem 2

Value at $T$ does first order taylor series
for $T$ induce error of 10%, 1%

\[ Y = \frac{1}{(1 - \beta^2)^{1/2}} \quad \text{Taylor series expansion} \]
\[ Y_T = 1 + \frac{1}{2} \beta^2 \]

What $\beta$ gives \( \frac{\delta_T}{\delta} = 0.1, 0.01 ? \)

Use Mathematica to solve

\[ \Theta T \beta \text{ for } 10\% \text{ error } 0.684 \]
\[ 1\% \approx 0.398 \]

Problem 2

\[
\text{NSolve}\left[ \frac{1}{\sqrt{1 - \beta^2}} \left( 1 + \frac{1}{2} \beta^2 \right) = \frac{\text{error}}{\sqrt{1 - \beta^2}}, \beta \right] / \text{. (error} \to 0.01) \\
\{(\beta \to -0.398428), (\beta \to 0.398428), (\beta \to 0. - 0.409441 \, \text{i}), (\beta \to 0. + 0.409441 \, \text{i}), (\beta \to 0. - 1.72948 \, \text{i}), (\beta \to 0. + 1.72948 \, \text{i})\}
\]

\[
\text{NSolve}\left[ \frac{1}{\sqrt{1 - \beta^2}} \left( 1 + \frac{1}{2} \beta^2 \right) = \frac{\text{error}}{\sqrt{1 - \beta^2}}, \beta \right] / \text{. (error} \to 0.1) \\
\{(\beta \to -0.684195), (\beta \to 0.684195), (\beta \to 0. - 0.746897 \, \text{i}), (\beta \to 0. + 0.746897 \, \text{i}), (\beta \to 0. - 1.70595 \, \text{i}), (\beta \to 0. + 1.70595 \, \text{i})\}
\]
\[
ke = m_0 c^2 (\gamma - 1)

\Rightarrow m_0 c^2 \gamma = 2m_0 c^2

\gamma = 2 \Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}

\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}

\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \Rightarrow v^2 = c^2 \left( \frac{3}{4} \right)

v = c \sqrt{\frac{3}{4}}
SPEED OF e⁻ \( KE = 3.4 \text{ keV} \)

\[ KE = m_0 c^2 (\gamma - 1) \]

\[ \frac{KE + m_0 c^2}{m_0 c^2} = \gamma \]

\[ \left( \frac{KE + m_0 c^2}{m_0 c^2} \right)^2 = \frac{1}{1 - \beta^2} \]

\[ 1 - \beta^2 = \frac{1}{\left( \frac{KE}{m_0 c^2} + 1 \right)^2} \]

\[ 1 - \frac{1}{\left( \frac{KE}{m_0 c^2} + 1 \right)^2} = \beta^2 \]

\( \frac{KE}{m_0 c^2} < 1 \)

\( \text{USE TAYLOR SERIES} \)

\( \frac{KE}{m_0 c^2} = \alpha \)

\( 1 + \alpha \approx 1 + n\alpha \)

\( n = -2 \)

\( \alpha = \frac{KE}{m_0 c^2} \)

\[ 1 - \left[ 1 - \frac{2\cdot KE}{m_0 c^2} \right] = \beta^2 \]

\[ \sqrt{\frac{2\cdot KE}{m_0 c^2}} = \beta = \frac{2\cdot 3.4 \text{ keV}}{511,000 \text{ keV}} = 0.06 \]

\[ \frac{2\cdot KE \text{ [keV]}}{511,000} \cdot \beta^2 = \left( \frac{v}{c} \right)^4 \]

\[ v = 0.06c \]

\[ \Rightarrow 1 \text{ MEV} = 0.25c \text{ not a good approximation} \]

\[ 10 \text{ keV} = 0.44c \text{ is that either} \]
Problem 5

CERN

\[ \begin{align*}
450 \text{ km} & \quad \text{60 ns ahead of light} \\
168,000 \text{ c} & \quad \Delta t \text{ ahead of c} \\
\end{align*} \]

SN1987A

\[ \begin{align*}
168,000 \text{ c} & \quad \Delta t \\
\end{align*} \]

\[ \frac{450 \text{ km}}{168,000 \text{ c}} = \frac{6 \times 10^{-8} \text{s}}{\Delta t} \]

\[ \Delta t = \frac{6.8 \times 10^{-8} \text{s}}{4.5 \times 10^{-5} \text{ m}} \]

\[ \Delta t = 2.2 \times 10^8 \text{ s} \]

\[ = 2.2 \times 10^8 \times \frac{1}{3 \times 10^{-7} \text{ y/y}} = 7.6 \text{ years} \]
Extra Credit Problem

SN 1987A

\[ 168,000 \text{ Ly} = 168,000 \text{ y} \cdot c \]

\[ \rightarrow \quad D \quad \rightarrow \]

\[ \text{EARTH} \]

\[ \text{Light} \]

\[ \nu \rightarrow \quad \text{\(\Delta T = 1 \text{ h}\)} \quad \rightarrow \]

\[ HE = 10 \text{ MeV} = m_v (\gamma - 1) c^2 \]

\[ T_{\text{light}} = \frac{L}{c} \]

\[ T_{\nu} = \frac{L}{\nu} \]

\[ \Delta T = T_{\nu} - T_L \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{L^2}{c^2}}} \]

\[ \Delta T \nu c = L \nu - \frac{L}{c} \]

\[ = \frac{L c - L \nu}{\nu c} \]

\[ = \frac{L c - L \nu}{\nu c} \]

\[ \Delta T \nu c = L c - L \nu \]

\[ \nu \Delta T c + L \nu = L c \]

\[ \nu (c \Delta T + L) = c \cdot L \]

\[ \frac{\nu}{c} = \frac{L}{c \Delta T + L} \]

\[ \frac{\nu}{c} = \frac{L}{c \Delta T + L} = \frac{1}{1 + \frac{c \Delta T}{L}} = (1 + \frac{c \Delta T}{L})^{-1} \]

\[ \text{real} \ (1 + \alpha)^n \approx 1 + n \alpha \]

\[ \alpha \ll 1 \quad \Rightarrow \quad n \ll 1 \]

\[ n = \gamma \quad \alpha = \frac{c \Delta T}{L} \]

\[ \frac{\nu}{c} = 1 - \frac{c \Delta T}{L} = \beta \]
Now find \( \gamma \)

\[
\gamma = \left(1 - B^2\right)^{-1/2}
\]

\[
= \left(1 - \left(1 - \frac{c\sigma T}{L}\right)^2\right)^{-1/2}
\]

\[
= \left(1 - \left(1 - \frac{2c\sigma T}{L} + \frac{c^2\sigma^2 T^2}{L^2}\right)\right)^{-1/2}
\]

\[
= \left(1 - 1 + \frac{2c\sigma T}{L} - \frac{c^2\sigma^2 T^2}{L^2}\right)^{-1/2}
\]

\[
= \left(\frac{c\sigma T}{L}\right)^{-1/2}
\]

\[
\gamma = \left(\frac{L}{c\sigma T}\right)^{1/2}
\]

\[
\text{KE} = m_0 \left(\sqrt{\frac{L}{2c\sigma T}} - 1\right) c^2
\]

\[
m_0 = \frac{\text{KE}}{\sqrt{\frac{L}{2c\sigma T} - 1}}
\]

\[
c^2 = \frac{10 \text{ MeV}}{168,000 \times 365.25 \times 24 \times 3600 \text{ seconds}}
\]

\[
= \frac{3.7 	imes 10^{-6} \text{ MeV}/c^2}{3.7 \times 10^6 \text{ MeV}/c^2}
\]

\[
= \frac{1}{3.7 \times 10^3 \text{ MeV}/c^2}
\]