Black Body Radiation
and
the experimental basis
for Quantum Theory

Max Plank
1858-1947

Nobel Prize in Physics 1918
Josef Stefan (1835-1893)

- Total energy radiated proportional to $T^4$

Question:
Suppose temperature raised from 300K (room temp) to 6000K (sun)

By what factor does the energy radiated increase?

A. 16
B. 16,000
C. 90,000
D. 160,000
Josef Stefan (1835-1893)

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Suppose temperature raised from 300K (room temp) to 6000K (sun)

By what factor does the energy radiated increase?

A. 16
B. 16,000
C. 90,000
D. 160,000  Answer: $(6000/300)^4 = 20^4 = 160,000$
Steam Radiator  400K   IR
Very Hot Stove   900K   Dull Red (transition to visible)
Lava               ~2000K   Glowing (most radiation still in IR)
Sun                 6000K   Most radiation visible
Wilhelm Wien (1864-1928)  
Nobel Prize 1911  

\[ \lambda_{\text{max}} = \left(2.90 \times 10^{-3} \text{ m} \cdot \text{K}\right)/T \]
Example of Black Body Spectra for different temperatures
Below is a photo of three stars. The light emitted by these stars is thermal radiation. Which of these stars is the hottest? The coolest?

A. Red, yellow
B. Red, blue
C. Yellow, blue
D. Blue, red
E. Blue, yellow
Below is a photo of three stars. The light emitted by these stars is thermal radiation. Which of these stars is the hottest? The coolest?

A. Red, yellow
B. Red, blue
C. Yellow, blue
D. Blue, red
E. Blue, yellow
Spectrum of high energy particle radiation from space does not follow a black body spectrum.

Above $10^{20}$ eV
Very low flux
~ 1 particle per km$^2$/sr/century

1 Joule
What is the best known example of a black body source?
What is the best known example of a black body source?

Hint  
Temperature = 2.7 K
Cosmic Microwave Background (Radiation from Big Bang!\nT=2.725K. The theoretical curve obscures the data points and the error bars.
Black Body Radiation and the experimental basis for Quantum Theory

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What is the energy of a “quanta” of RED light?

660 nm wavelength in units of electron volts?

\[ E = h \nu \]

\[ h = 6.62 \times 10^{-34} \text{ Js} \]
What is the energy of a “quanta” of RED light? 660 nm wavelength in units of electron volts?

\[ E = h \nu \]

\[ h = 6.62 \times 10^{-34} \text{ Js} \]
\[ \nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{660 \times 10^{-9} \text{ m}} = 4.54 \times 10^{14} \text{ s}^{-1} \]

\[ E = 6.62 \times 10^{-34} \text{ Js} \times 4.54 \times 10^{14} \text{ s}^{-1} \]
\[ E = 3.01 \times 10^{-19} \text{ J} \]

\[ 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} \]
\[ E = \frac{3.01 \times 10^{-19} \text{ J}}{1.602 \times 10^{-19} \text{ J/eV}} \]
\[ E = 1.88 \text{ eV} \]
What is the energy of a “quanta” of RED light?
660 nm wavelength in units of electron Volts

Easier Way to Solve this

\[ E = \frac{hc}{\lambda} \]
\[ hc = 1240 \text{ eV nm} \]
(useful constant to remember)

\[ E = \frac{1240}{660} \text{ eV} \]
\[ E = 1.88 \text{ eV} \]
Question: What is the energy quantization of a grandfather clock?

Hint: \( \hbar = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} \)
Question:
What is the energy quantization of a grandfather clock?

Hint:
\[ \hbar = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \]
\[ E = n \hbar \nu \]
for n=1, \( \nu = 1 \text{ Hz} = 1 \text{s}^{-1} \)
\[ E = 6.6 \times 10^{-34} \text{ J} \]
How is the quantization realized?

\[ E = n \hbar \nu \]

for \( n=1 \), \( \nu = 1\text{Hz} = 1\text{s}^{-1} \)

\[ E = 6.6 \times 10^{-34} \text{J} \]
How does this quantization translate into quantization of the pendulum displacement (height)?

\[ E = nhv \]
for \( n=1 \), \( v=1\text{Hz}=1\text{s}^{-1} \) \( E = 6.6 \times 10^{-34} \text{J} \)
How does this quantization translate into quantization of the pendulum displacement (height)?

\[ E = n\hbar \nu \]

For \( n = 1 \), \( \nu = 1 \text{Hz} = 1\text{s}^{-1} \)

\[ E = 6.6 \times 10^{-34} \text{J} \]

\[ E = mgH = 6.6 \times 10^{-34} \text{J} \]

\[ H = \frac{6.6 \times 10^{-34} \text{J}}{(1 \text{kg} \times 10 \text{m/s}^2)} = 6.6 \times 10^{-35} \text{m} \]

Too small to measure (size of an atom is about \( 10^{-8} \text{ m} \))
Wien’s displacement law

- The intensity $\mathcal{L}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature.
- **Wien’s displacement law**: The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.

Visible light: 400 – 700 nm  
UltraViolet: <400 nm  
Infrared: >700 nm

\[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K} \]

Empirical Formula

Wilhem Wien: Nobel Prize 1911
Understanding the blackbody radiation spectrum

- Attempts to fit the low and high wavelength part of the spectrum

- Using classical theory of electromagnetism and thermodynamics, Lord Rayleigh comes up with:
  \[
  J(\lambda, T) = \frac{2\pi c k T}{\lambda^4}
  \]
  Rayleigh-Jeans formula

- Major flaw at short wavelength ("Ultraviolet catastrophe")

Describing the blackbody emission spectra: one of the outstanding problems at the beginning of the 20th century
Two Catastrophes?

• Classical physics:
  – Emission spectrum: a superposition of electromagnetic waves of different frequencies
  – Frequencies allowed: standing waves inside the cavity

• Equipartition of the energy:
  – Every standing wave carries $kT$ of energy
  – Flaw: when $\lambda \to 0$, the number of standing waves $\to \infty$, leading to $E \to \infty$ [Ultraviolet Catastrophe]

• Failure of classical theories:
  – The work of Rayleigh-Jeans was considered as state-of-the-art, using well tested theories, which were in very good agreement with experimental results in many other circumstances.
  – Need for a new theory…
Planck’s radiation law

- Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of “oscillators” contained in the walls. He used Boltzmann’s statistical methods to arrive at the following formula:

\[
I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}
\]

Planck’s radiation law

- Planck made two modifications to the classical theory:
  1) The oscillators (of electromagnetic origin) can only have certain discrete energies determined by \( E_n = nh\nu \), where \( n \) is an integer, \( \nu \) is the frequency, and \( h \) is called Planck’s constant.

\[
h = 6.6261 \times 10^{-34} \text{ J} \cdot \text{s}
\]

2) The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

\[
\Delta E = h\nu
\]
So what about objects and THERMAL RADIATION?
WHAT WAS KNOWN?

HOTTER OBJECTS EMIT MORE ENERGY,
HOW MUCH MORE? PROPORTIONAL TO \((\text{TEMP})^4\)

\[ E (\text{heat, light}) \]

\[ \text{INCREASE } T \]
\[ \text{INCREASE } E \propto T^4 \]

or specifically

\[ I = \sigma T^4 \]

\[ \sigma = 5.67 \times 10^{-8} \frac{\text{Watts}}{\text{m}^2 \text{K}^4} \]

\[ I \text{ emissivity watts radiated} \]

\[ (k \text{ is Temp in kelvin}) \]

(One homework problem is to calculate energy radiated by a person)

(Note the \(T^4\) is important)

Suppose you have an object at room temperature + heat it to 6000K
How much more energy is radiated?

\[ \frac{I_{6000}}{I_{300}} = \frac{\sigma (6000)^4}{\sigma (300)^4} = (20)^4 \leq 60,000 \]

\[ 10^4 \times 2^4 = 1.6 \times 10^5 \]

60,000 TIMES MORE ENERGY

NAME AN OBJECT AT 60, 6000K? SUN!!
Was also known that not just the amount of radiation changes, but also the nature of radiation changes

⇒ color changes when something gets hot
  Black → Red → Yellow → White

Hotter objects ⇒ shorter wavelengths

Question: Could this be measured/calculated?
⇒ first need something called a perfect black body

Was known for some time that materials that were good absorbers were also good emitters & vice versa.

Was there a material that was a perfect absorber (a body be light at all frequencies)?

Then if it was heated, it would emit light at all frequencies. Not the same amount at every frequency, but the distribution would not be distorted by some feature specific to the shape or texture of the object.

How to make a "black body"?

Drill a hole in a hollow box

Light can get in, but it can never leave, sort of a "Hotel California" for light (Wilhelm Wien 1864 - 1928)

Note: This device can be made from any material.

Better to have a dark material, but even a white material works
So Wien made cavity blackbody, heated it and measured the distribution of light emitted.

\[
\text{Light} \xrightarrow{\text{Prism}} \text{Light Separated by Wavelength}
\]

The one thing he measured was the most probable wavelength, i.e., the wavelength at which more light was emitted than any other.

Found:
\[
\lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \mu}{T}
\]

This has applications →

Given color of something, you can figure out temperature of object.

→ Stars → Measure spectrum, find most probable \( \lambda \), then use equation above to find \( T \)

Then use \( I = \sigma T^4 \) to find emissivity.

Wien also measured distribution of emitted light, how much light vs \( \lambda \)?
But before discussing the measured distribution, I will first discuss what was predicted. Prediction made by Lord Rayleigh (1842–1919) who was a well respected scientist with a number of successful predictions.

For example, using Maxwell's equations, he predicted that the scattering of light in air would be $\propto \frac{1}{\lambda^4}$ which is true (or nearly so). This explains, for example why sunsets are red and the sky is blue.

So that was one example of a successful prediction by Rayleigh.

However...
RAYLEIGH MODEL

CARTOON FORM

BLACK BODY

LARGE $\lambda$
Few "modes"

SMALL $\lambda$, many modes.
(Easy to add 1 more antinode)

100, 1001, 102, 103 \ldots

So MANY higher energy modes

This was the origin
of the UV catastrophe

RAYLEIGH DID THIS MUST DO
in 3D

GOT \quad S(\lambda) \propto \frac{1}{\lambda^4}
CLEAR DISAGREEMENT BETWEEN DATA AND CLASSICAL PREDICTION AS \( \lambda \to 0 \), THEORY PREDICTS INFINITE AMOUNT OF ENERGY RADIATED AT SHORT \( \lambda \) (large \( \nu \))

Resolved by Max Planck (1858-1947)

**Insight:**
1. Energy Flow (Energy radiated) **NOT** continuous. **Discrete**!
2. Energy comes in small packets or bits, Planck called them **QUANTA**
3. Size of these energy packets (amount of energy in a packet) is not the same for different wavelengths. Size of packet \( \propto \nu \) (frequency) i.e., violet packet is larger than a red packet (\( \nu_{\text{violet}} > \nu_{\text{red}} \))
4. Probability for a Black body to emit a quanta is less for a large quanta and more for a small quanta. Although Planck did not know the mechanism for radiating a quanta, he hypothesized that energy was accumulated (somehow) in the Black body. To emit a large quanta, more energy had to be accumulated. So chances were higher that since a lower energy (red) quanta
would be emitted first, since that smaller amount of energy would be accumulated first, so higher probability to emit large \( \lambda \) (small \( v \)) quanta, and less probability to emit small \( \lambda \) (large \( v \)) packet (quanta) 

\[ \Rightarrow \text{less energy radiated at small } \lambda \text{ (large } v) \]

\[ \Rightarrow \text{AVOID THE UV CATASTROPHE} \]

A rough analogy \[ \Rightarrow \text{consider a student wants to get a new laptop/bike/car. This requires accumulating a relatively large amount of } \$\text{. But while trying to accumulate this large sum, many smaller expenses come up - food, clothes, cell phone bill, tickets... and it is much more probable that } \$\text{ will be spent (emitted) for the in small packets for these items. In contrast the probability the student will accumulate funds to purchase a Ferrari are essentially zero.} \]
Planck predicted quantization of energy

\[ E = n \hbar \nu \]

\( \downarrow \) Planck's constant

\( h = 6.6 \times 10^{-34} \text{ J s} \)

Also very useful \( \hbar c = 1240 \text{ eV nm} \)

\( \Rightarrow \) Example: How big are quanta emitted by He Ne Laser?

Laser Pointer \( \lambda = 662 \text{ nm} \)

\[ E = h \nu = \frac{\hbar c}{\lambda} = \frac{1240 \text{ eV nm}}{662 \text{ nm}} = 1.88 \text{ eV} \]

Suppose laser emits 1 mW? How many quanta per second

Power = 1 mW = \( 10^{-3} \) Joule/second

\[ 10^{-3} \frac{J}{s} = 10^{-3} \times 6 \times 10^{18} \frac{\text{eV}}{\text{sec}} \times 1 \frac{\text{quantum}}{1.88 \text{ eV}} \approx 2 \times 10^{15} \frac{\text{quantum}}{\text{sec}} \]

or \( 2 \times 10^{15} \) photons/second
Quantization of Clock (Pendulum)

\[ E = mgh = n\hbar\nu \]

\[ H = \frac{n\hbar\nu}{mg} = \frac{1 \times 6.6 \times 10^{-34} J \cdot s}{1 \text{ kg} \times 10 \text{ m/s}^2} = 1.6 \times 10^{-33} \text{ m} \]

Can this be measured?

No size of atom \( \sim 10^{-8} \text{ m} \)

Much too small to see for massive, slow object.
So Planck obtained

\[ S(\lambda) = \frac{8\pi}{\lambda^4} \frac{c}{4} \frac{hc}{\lambda} \left( \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} \right) \]

\[ n = 6.6 \times 10^{-24} \text{ J.s} \quad \text{Planck's constant} \]

\[ k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J/K} \]

Can derive several things from this equation.

When \( \lambda \) large you get the classical expression \( \frac{8\pi}{\lambda^4} \frac{c}{4} kT \)

Trick is to expand \( \frac{1}{e^{\frac{hc}{kT\lambda}} - 1} \) part using \( e^x = 1 + x + \frac{1}{2} x^2 + \ldots \)

\[ \left( \frac{1}{2 + \frac{hc}{kT\lambda}} - 1 \right) \approx -\frac{kT}{hc} \]

\[ S(\lambda) = \frac{8\pi}{\lambda^4} \frac{c}{4} \frac{hc}{\lambda} \left( \frac{kT}{hc} \right) \]

\[ \approx \frac{8\pi}{\lambda^4} \frac{c}{4} kT \]

which is classical expression.
Can derive Wien's law \( \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{mK} \)

\[
\frac{dS(\lambda)}{d\lambda} = 0 = \frac{8\pi c}{4} \frac{hc}{\lambda^2} \left( \frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \right) = 0
\]

\[
= -5 \frac{1}{\lambda^6} \frac{1}{e^x} = \frac{1}{\lambda^5} \frac{-hc}{kT \lambda^2} \frac{e^{\frac{-hc}{kT \lambda}}}{(e^{\frac{-hc}{kT \lambda}} - 1)}
\]

Multiply by \( \lambda^5(e^{\frac{-hc}{kT \lambda}} - 1) \)

\[
= -\frac{5}{2} + \frac{hc}{kT \lambda^2} \frac{e^{\frac{hc}{kT \lambda}}}{(e^{\frac{-hc}{kT \lambda}} - 1)} = 0
\]

define \( x \equiv \frac{hc}{kT \lambda} \)

\[
5 \frac{x e^x}{e^x - 1} = 0
\]

transcendental Eqn solution is \( x = 4.966 \)

\[
4.966 = \frac{hc}{kT \lambda_{\text{max}}}
\]

\[
\lambda_{\text{max}} T = \frac{hc}{k \cdot 4.966} = \frac{6.6 \times 10^{-34} \text{ J.s}}{1.38 \times 10^{-23} \text{ J/K}} \cdot 4.966
\]

\[
\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ mK}
\]