Compton Scattering

Arthur Compton 1892-1962
Nobel Prize in Physics 1927
Suppose that you do the following Experiment.

What might you expect for $\lambda_{\text{final}} - \lambda_{\text{initial}}$

1. $\lambda_{\text{final}} - \lambda_{\text{initial}} > 0$
2. $\lambda_{\text{final}} - \lambda_{\text{initial}} = 0$
3. $\lambda_{\text{final}} - \lambda_{\text{initial}} < 0$
Suppose that you do the following Experiment.

What might you expect for $\lambda_{\text{final}} - \lambda_{\text{initial}}$?

1. $\lambda_{\text{final}} - \lambda_{\text{initial}} > 0$  \textbf{Compton Scattering}
2. $\lambda_{\text{final}} - \lambda_{\text{initial}} = 0$  \textbf{(Classical Prediction – describes visible light)}
3. $\lambda_{\text{final}} - \lambda_{\text{initial}} < 0$  \textbf{NO (higher energy gamma ray emitted)}

Suppose you change the angle?
Suppose that you do the following experiment.

Light \( \lambda_{\text{init}} \) (Gamma Rays)

Piece of Stuff

Assuming the emitted wavelength changes,

Which would you guess to be closer to \( \lambda_{\text{init}} \)?

A or B?
Suppose that you do the following experiment.

Assuming the emitted wavelength changes...

Which would you guess to be closer to $\lambda_{\text{init}}$?

Consider the case where $\theta$ approaches 0.
THE SCATTERING OF X-RAYS.*

BY

ARTHUR H. COMPTON, Ph.D.

Professor of Physics, Ryerson Physical Laboratory, University of Chicago.

The present paper will be confined to a discussion of some of the points which present to us a revolutionary change in our ideas regarding the process of scattering of electromagnetic waves. No attempt will be made to describe the great amount of experimental and theoretical work which has been done on

Within the past two years, however, scattering phenomena have been observed which are so directly contrary to the predictions of the usual electrodynamics that we apparently have to reverse our attitude almost completely. We thought we could explain the scattering of X-rays on the assumption that radiation proceeds in spherical waves, spreading in all directions in space. We now find that to retain this assumption, if our recent results are correct, we must abandon both the principle of the conservation of momentum and the principle of the conservation of energy—a hard choice, indeed; but that our observations are explained simply if we are willing to imagine the rays as consisting of discrete quanta proceeding in definite directions.
A thin beam of X rays approaches target...

...and is deflected (scattered) in several directions by target.

Detector measures X rays emerging at an angle $\theta$.

The spectrum of the rays from the radiator $R$, at an angle $\phi$ with primary beam is investigated.
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**PHGN 326  Advanced Lab II**  
**Compton Scattering Experiment**

- **137Cs Source**
- **Gamma Ray Source (0.66 MeV)**
- **Aluminium Post**
Compton Scattering Experiment
PHGN 326  Advanced Lab II

\[ \lambda' - \lambda = \left( \frac{h}{cm_e} \right) (1 - \cos \theta) \]

137Cs Source
Gamma Ray Source
(E=0.66 MeV)

Aluminium Post

Gamma Ray Detector
(Measure E)
Scattering Energy Vs. Angle

Energy (eV)

Angle (deg)

Caption: What are the two data points sets?
The laboratory measurements of Compton Scattering are well described by the theoretical predictions.

What pieces of fundamental physics does this validate?
The laboratory measurements of Compton Scattering are well described by the theoretical predictions. What pieces of fundamental physics does this validate?

Quantization of light
Light can be described as a particle
Energy-Mass relationship (Conservation of total energy)
Limits on light-speed anisotropies from Compton scattering of high-energy electrons


1LPSC, UJF Grenoble 1, CNRS/IN2P3, INPG, 53 avenue des Martyrs 38026 Grenoble, France
2INFN Sezione di Roma TV, 00133 Roma, Italy
3INFN Sezione di Catania and Università di Catania, 95100 Catania, Italy
4INFN Laboratori Nazionali di Frascati, 00044 Frascati, Italy
5INFN Sezione di Roma TV and Università di Roma “Tor Vergata,” 00133 Roma, Italy
6INFN Sezione di Torino and Università di Torino, 10125 Torino, Italy
7INFN Sezione di Roma I and Istituto Superiore di Sanità, 00161 Roma, Italy
8INFN Sezione di Catania and Università di Messina, 98166 Messina, Italy
9Yerevan Physics Institute, 375036 Yerevan, Armenia
10Yerevan State University, 375025 Yerevan, Armenia
11Institute for Nuclear Research, 117312 Moscow, Russia
12ICN, Universidad Nacional Autónoma de México, A. Postal 70-543, 04510 México D.F., Mexico

(Dated: June 10, 2010)

The possibility of anisotropies in the speed of light relative to the limiting speed of electrons is considered. The absence of sidereal variations in the energy of Compton-edge photons at the ESRF’s GRAAL facility constrains such anisotropies representing the first non-threshold collision-kinematics study of Lorentz violation. When interpreted within the minimal Standard-Model Extension, this result yields the two-sided limit of $1.6 \times 10^{-14}$ at 95% confidence level on a combination of the parity-violating photon and electron coefficients $(\kappa_{\nu})^{Y/Z}$, $(\kappa_{\nu})^{Z/X}$, $\epsilon_{TX}$, and $\epsilon_{TY}$. This new constraint provides an improvement over previous bounds by one order of magnitude.

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Compton Scattering

Named after Author Compton 1892-1962 1927 Nobel Prize

Continue discussion of interaction of light and matter + the quantum nature of light

→ PH326 (Ad Lab II) we do a Compton Scattering Experiment!

Compton worked with high energy light (Photons) with energies in the keV → 10s keV energy range

(PH 326 we use radioactive sources that produce γ rays through nuclear decay, specifically 137Cs)

Compton obtained γ rays from e⁻ → metal or e⁻ → Nucleus (e+ charge) → γ Bremstrahlung Effect

But this class is about what he did with the high energy photons

\[ \lambda_i < \lambda_f \]

Graphite

⇒ So How to understand this effect?

Classical Explanation

X-rays are high frequency waves

\[ \lambda_{\text{IN}} = \lambda_{\text{OUT}} \]

WORKS FOR VISIBLE LIGHT

Red → Red Laser pointer light is still RED!!

Quantum Explanation

higher \( E \) shorter \( \lambda \)
photon loses energy (to electron)

⇒ Treat Photons + Electrons as Particles!
Recall that to treat photons as particles with momentum and energy, we must use

\[ E^2 = p^2c^2 + m_e^2c^4 \quad \Rightarrow \quad E_{\text{photon}} = \frac{p\,c}{\text{photon}} \]

\[ = 0 \quad \text{for photons} \]

\[ E_{\text{photon}} = \frac{p}{\text{photon}} \, c = h \nu \quad \Rightarrow \quad P_{\text{photon}} = \frac{h \nu}{c} = \frac{h}{\lambda} \]

\[ \lambda \nu = c \]

For X-rays \( \lambda = 1 \text{ nm} \) (Red Light \( \lambda = 660 \text{ nm} \))

\[ E = \frac{hc}{\lambda} = 1240 \frac{eV\text{nm}}{1\text{nm}} = 1.24 \text{ keV} \]

Consider

\[ \lambda_f \quad \lambda_c \]

\[ \theta \]

Target (\( e^- \))

\[ \lambda_f - \lambda_c = \text{CONST.} \ (1 - \cos \theta) \]

If \( \theta = 180^\circ \), \( \cos(180^\circ) = -1 \)

\[ \lambda_f - \lambda_c = \frac{2h}{m_e c} \quad \text{note: does not depend on} \ \lambda ! \]

\[ \Rightarrow \text{CAN ALL THIS BE EXPLAINED USING QUANTA} \]

\[ + \text{CONS. OF } E, \ p, \ \text{Treat } e^- + \text{photons as PARTICLES.} \]
The general problem: \[ E_f, P_f, \lambda_f \]

Photon

\[ E, P, \lambda \]

What is \( \lambda_f - \lambda_i \)?

(e\(^-\) typically not observed. It gets absorbed in the target material)

Apply Conservation of Energy, Momentum

Energy

Photons \[ E_{\text{INIT}} = E_{\text{FINAL}} \]

\[ E + m_e c^2 = E_f + E_e \]

also have

\[ E_e^2 = m_e c^4 + P_e^2 c^2 \]

Momentum Conservation

\[ P_{x \text{INIT}} = P_{x \text{FINAL}} \quad P_{y \text{INIT}} = P_{y \text{FINAL}} \]

\[ P = P_f \cos \Theta + P_e \cos \Phi \quad \delta = P_f \sin \Theta - P_e \sin \Phi \]

need to eliminate \( P_e, E_e \), put \( e^- \) terms on \( R \)

\( A \) - \( P_f \cos \Theta = P_e \cos \Phi \)

\( B \) - \( P_f \sin \Theta = P_e \sin \Phi \)

Note \( P_e^2 \cos^2 \Phi + P_e^2 \sin^2 \Phi = P_e^2 \)

so square both sides of \( A, B \) and add

\[ (P - P_f \cos \Theta)^2 + P_f^2 \sin^2 \Theta = P_e^2 \cos^2 \Phi + P_e^2 \sin^2 \Phi = P_e^2 \]

expand this

\( C \) - \( P^2 - 2P P_f \cos \Theta + P_f^2 \cos^2 \Theta + P_f^2 \sin^2 \Theta = P_e^2 \)

To eliminate \( E_e, P_e \)

substitute \( 2, 3 \) into \( 1 \)

use \( 3 \) to eliminate \( P_e^2 \)

use \( 2 \) to get

\[ E_e^2 = P_e^2 c^2 + m_e^2 c^4 \]

use \( 2 \)

\[ (E + m_e c^2 - E_f)^2 = (P^2 - 2P P_f \cos \Theta + P_f^2)^2 c^2 + m_e^2 c^4 \]

Then expand and watch terms cancel! 1/1
\[ E^2 + E_m c^2 - E_E c^2 + m_e c^2 E_f = m_e c^2 E_f - E_E + m_e c^2 E_f + E_f^2 = \]

\[ (P^2 - 2P\cos \theta + P_f^2) c^2 + m_e c^2 E_f \]

\[ E^2 + E_m c^2 - E_E c^2 + m_e c^2 E_f - m_e c^2 E_f - E_E + m_e c^2 E_f + E_f^2 = \]

\[ P^2 c^2 - 2P\cos \theta + P_f^2 c^2 + m_e c^2 E_f \]

\[ E_{\text{photon}} = P_{\text{photon}} c \quad \text{works for initial and final photons} \]

\[ 2E_m c^2 - 2E_E c^2 - 2m_e c^2 E_f = -2P\cos \theta \]

2 will cancel \[ \frac{P}{E} = \frac{1}{c} \quad \frac{P_f}{E_f} = \frac{1}{c} \]

so divide both sides by \(2E_E c^2 m_e\)

\[ \frac{2E_m c^2 - 2E_E c^2 - 2m_e c^2 E_f}{2E_E c^2 m_e} = -2P\cos \theta \]

\[ \frac{E_f}{E} - \frac{1}{m_e c^2} - \frac{1}{E} = -\frac{1}{c} \frac{1}{m_e c} \cos \theta \]

\[ \frac{1}{E_f} - \frac{1}{E} = \frac{1}{m_e c^2} - \frac{1}{m_e c^2} \cos \theta \]

\[ E = \frac{hc}{\lambda} \rightarrow \frac{1}{E} = \frac{\lambda}{hc} \]

\[ \frac{\lambda_f}{hc} - \frac{\lambda}{hc} = \frac{1}{m_e c^2} (1 - \cos \theta) \]

\[ \lambda_f - \lambda = \frac{1}{m_e c} \left( \frac{h}{c} \right) (1 - \cos \theta) \]

multiply both sides by \(hc\)

what is \(\frac{h}{m_e c}\)?

\[ \frac{hc}{m_e c^2} = \frac{1240 \text{ eV nm}}{0.511 \text{ MeV} c^2} = \frac{1240 \text{ nm}}{511,000} \]

\[ \frac{h}{m_e c} = 0.00243 \text{ nm} \]
$$\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \Theta)$$

$\Theta = 0 \quad \cos(\Theta=0)=1 \Rightarrow \lambda_f - \lambda_i = 0$

$\Theta = 90 \quad \cos(\Theta=90)=0 \quad \lambda_f - \lambda_i = \frac{h}{m_e c}$

$\Theta = 180 \quad \cos(\Theta=180)=-1 \quad \lambda_f - \lambda_i = \frac{2h}{m_e c}$

so going back to example with the laser

what about red light. Was our
observation + the compton equation
consistent? \text{ YES } \Rightarrow \Delta \lambda = 0.00243

too small to see 2 parts in 100,000

But consider x rays \( \lambda = 0.01 \text{ nm} \)

\[
E = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{0.01 \text{ nm}} = 124000 \text{ eV} = 124 \text{ keV}
\]

\( \lambda_i = 0.01 \text{ nm} \)

\( \lambda_f = 0.01243 \text{ nm} \)

YOU COULD SEE THIS IS WHAT COMPTON DID

\[
\frac{E_f}{\lambda_f} = \frac{hc}{\lambda_f} = \frac{1240 \text{ eV nm}}{0.01243 \text{ nm}} = 99.8 \text{ keV}
\]

difference of \( \sim 25 \text{ keV} \)
So what have we done?

Shown that experimentally observed

\[ \lambda_f - \lambda_i = \frac{\hbar}{m_e c} (1 - \cos \Theta) \]

Can be derived using cons. of P, E, and treating both electrons + light as quanta!!
To Recap, we have considered three fundamental processes:

- **Black Body Planck**: Light as quanta, \( E = h \nu \), \( n = 1 \) for 1 quantum (photon)

- **Photo Electric Effect**: Einstein, Milliken
  \[
  \text{KE}_e = h \nu - W
  \]

- **Compton Scattering**: Quanta, S.R. Ems, Poms
  (treat photons, e- as particles)
  \[
  \lambda_f - \lambda_i = \frac{h}{mc} (1 - \cos \theta)
  \]

These three effects - measured carefully and described by theory invoking quanta - confirmed the quanta nature of light and electrons.
Final note on Photomultiplier Tubes (PMTs)

Glass envelope

Thin coating of photocathode material (metallic)

But doesn't Compton scattering happen too?

Ans. Yes.

Vacuum

Reflection!

Metal is reflective.
Use thin coating

$\Rightarrow$ So $e^-$ can escape for photoelectric effect

$\Rightarrow$ To reduce chances of reflection

Note for IR, VIS, UV light $\Delta x$ is small. But the reflection in the incident light is a contributor to the inefficiency of PMTs