More about Waves

Royal

Stadium

Tricky Wave
most simple case

\[ y_1(x,t) = A_1 \cos k_1(x - v_1 t) \]
\[ y_2(x,t) = A_2 \cos k_2(x - v_2 t) \]

;\]
\[ A_1 = A_2 = A \]
\[ v_1 = v_2 = v \]
\[ k_1 = k_2 = k \]

\[ y_{\text{TOTAL}} = 2A \cos k(x - vt) \]

\[ \rightarrow \text{constructive interference} \]
Beats: superposition of two waves of slightly different frequency: they interfere to give a pulsating amplitude.

\[ y(t) = y_1(t) + y_2(t) = A \left( \cos(2\pi v_1 t) + \cos(2\pi v_2 t) \right) \]
\[ = 2A \cos 2\pi \left( \frac{v_1 - v_2}{2} \right) t \cdot \cos 2\pi \left( \frac{v_1 + v_2}{2} \right) t \]

Low Frequency ("beat frequency")  “High” Frequency [Applet]
Standing Waves (I)

- Boundary conditions: $y(x=0) = y(x=L) = 0$
- The wave pattern doesn’t appear to move, only Amplitude change
- Node (N): point of zero displacement
- Anti-Node (A): point of maximum displacement
Standing Waves (II)

- Traveling wave \( y_1(x,t) + \) Reflected Wave \( y_2(x,t) \) 
  \[= y(x,t) \] [Harmonic waves]
- Boundary conditions:
  - \( y_1(x,t) = A \cos(kx - \omega t) \)
  - \( y_2(x,t) = -A \cos(kx + \omega t) \) [Newton’s 3\(^{rd}\) law]

- \( y(0,t) = A \cos(-\omega t) - A \cos(+\omega t) = 0 \)
- \( y(L,t) = A \cos(kL - \omega t) - A \cos(kL + \omega t) = 0 \)
  \[\Rightarrow y(L,t) = 2A \sin(kL) \sin(\omega t) = 0 \]
  \[\Rightarrow \sin(kL) = 0 \Rightarrow k_n = n\pi/L \]
Standing Waves (III)

\[ y(x,t) = y_1(x,t) + y_2(x,t) = A \left( \cos(k_n x - \omega t) - \cos(k_n x + \omega t) \right) \]
\[ = A \left( \cos((n\pi/L)x - \omega t) - \cos((n\pi/L)x + \omega t) \right) \]
\[ = 2A \sin((n\pi/L)x) \sin(\omega t) \]

Nodes:
\[ y(x_{\text{node}},t)=0 \quad \Rightarrow \quad \sin((n\pi/L)x_{\text{node}})=0 \]
\[ \Rightarrow \quad (n\pi/L)x_{\text{node}}=m\pi \quad \text{for} \quad 0 \leq x \leq L \]
\[ \Rightarrow \quad x_{\text{node}} = mL/n \quad (m\leq n) \]

With \( k = 2\pi/\lambda \Rightarrow \lambda_n = 2\pi/k_n = 2L/n \)
\[ \Rightarrow \quad x_{\text{node}} = m\lambda_n/2 \quad (m\leq n) \]
Dude…
it's a wave.

Is light a wave?
Bart says YES and so
does Thomas Young.

Dude...
it’s a wave.
No, it’s particles
parallel wave crests

light from distant source

wavelength ($\lambda$)

2 Slits
parallel wave crests

light from distant source

wavelength (λ)
parallel wave crests

light from distant source

wavelength (\(\lambda\))

constructive
destructive
Path lengths difference: $\Delta L = d \sin \theta$

(Assumption: $\theta \approx \theta'$ correct, if $L \gg d$)
For the light coming from $S_2$ to be “in phase” with the light coming from $S_1$: $\Delta L = n\lambda$

\[
y = L \tan \theta \ ; \ \theta \text{ small, } \tan \theta \approx \sin \theta \Rightarrow y = L \sin \theta
\]

\[
\Delta L = n\lambda = d \sin \theta
\]

\[
y_n = \frac{nL\lambda}{d}
\]

Position of $n^{th}$ bright fringe

“out of phase” $\Rightarrow \Delta L = (n+\frac{1}{2})\lambda$  \[
y_n = \frac{(n+\frac{1}{2})L\lambda}{d}
\]

Position of $n^{th}$ dark fringe
Young’s Double Slit Experiment (1802)

T. Young
1773-1829

YouTube Demo
http://www.youtube.com/watch?v=UANVMlajqllA
Electromagnetic (light) wave

Picture of an e.m. (linearly polarized) wave

- Transverse waves
- Transport energy (and momentum)
- Can travel through vacuum (!) and certain solids, liquids and gases
- Do not transport matter or charge
- From the Maxwell’s Equations: wave equation with $c_{\text{vacuum}} = 1/\sqrt{\varepsilon_0\mu_0}$
Single slit diffraction: Analysis (I)

- Huygens principle:
  - Waves 1 & 3 (and 3 & 5) out of phase:
    \[(a/2) \sin \theta_1 = \lambda/2\]
Single slit diffraction: Analysis (II)

- **Huygens principle:**
  - Waves 2 & 3 (and 1 & 2, 3 & 4, 4 & 5) out of phase:
    \[(a/4) \sin \theta_2 = \lambda/2\]
  - Generalization:
    \[(a/2n) \sin \theta_n = \lambda/2\]
- **Dark fringes in single slit diffraction:**
  \[\sin \theta_n = n\lambda/a\] with \(n = \pm 1, \pm 2, \pm 3, \ldots\)
Diffraction Examples

Diffraction due to a sharp (straight) edge

Light Diffraction by a Razor Blade

“Single slit” diffraction with your 2 thumbs!
Multiple Slits & Diffraction Grating

Diffraction grating:
hundreds or thousands of slits

\[ d \sin \theta \]

\[ P \]

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(a) \( N = 2 \)

(b) \( N = 8 \)

(c) \( N = 16 \)
Application: grating spectrograph

1. Light from telescope is sent along fiber-optic cables (not shown) and emerges here

2. Light strikes concave mirror and emerges as a beam of parallel rays

3. Light passes through diffraction grating

4. Lenses direct diffracted light onto a second concave mirror

5. Concave mirror reflects light to a focus

6. An electronic detector (like the one in a digital camera) records the spectrum

Diffraction grating:
About 1000 slits per mm

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Optical Properties

- **Interference**: the superposition of two light beams to produce alternating regions of bright light and dim light

- **Diffraction**: the spreading of waves around corners (light waves, water waves)
  - One convincing proof of the wave nature of light: contradicts geometric optics

- **Huygens principle**: every point on a wave front of light may be a source of secondary wave fronts / wavelets

- **Coherence**: two sources of light with the same frequency and a constant phase difference are called coherent

- **Monochromatic**: “one color”, only one wavelength/frequency
Doppler Effect: Fixed Source, Moving Listener

Source: Wavelength: $\lambda = \frac{v}{v_s}$ with $v_s$, frequency at the source $S$

Listener: Relative velocity of the wave front $= v_L + v$
   
   Same wavelength, but now different frequency: $v_L = \frac{(v_L + v)}{\lambda}$
   
   $\lambda = \frac{v}{v_s}$

$\Rightarrow v_L = \left( \frac{(v_L + v)}{v} \right) v_s$
Doppler Effect: Moving Source, Moving Listener

Source: moving at velocity $v_s$. During one cycle: $T_s = 1/v_s$, the wave travels a distance of: $v T_s = v/v_s$, while the source travels $v_s T_s = v_s/v_s$.

Wavelength (distance between two crests):
- In front of the source: $\lambda = (v / v_s - v_s / v_s)$
- Behind the source: $\lambda = (v / v_s + v_s / v_s)$

Listener: Relative velocity of the wave front = $v_L + v$

$v_L = (v_L + v) / \lambda$ with $\lambda = (v + v_s) / v_s$

$\Rightarrow v_L = \left( \left( \frac{v_L + v}{v_s + v} \right) \right) v_s$
Last time simple wave \( y(x,t) = A \cos k(x - vt) \)

and wave equation \( \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{v^2}{2} \frac{\partial^2 y}{\partial x^2} \) and physical meaning/interpretation

Today - interference effects - combine waves

Why? important property of waves

Wave vs particle description of matter

Probe of distance \( \lambda \sim 500 \text{nm} \) \( 5 \times 10^{-7} \text{m} \)

Applications: interferometers, Quantum Mechanics, fundamental physics

Most simple case

\[ y_1(x,t) = A_1 \cos k_1(x - v_1 t) \]
\[ y_2(x,t) = A_2 \cos k_2(x - v_2 t) \]

\[ v_1 = v_2 = v \]
\[ k_1 = k_2 = k \]

\[ y_{\text{Total}} = 2A \cos k(x - vt) \]

→ constructive interference
Other examples

Destructive interference?

\[ y_2(x, t) = A \cos k(x - Vt) + \pi \]

\[ \uparrow \]

Phase shift

suppose \( V_1 = V_2 \), \( k_1 \neq k_2 \) (but close)

\[ \rightarrow \text{beat frequency} \quad (\text{Homework problem}) \]

Stationary observer

source moving away at some velocity

Doppler Shift observer measures longer \( \lambda \)

(red shift)

source moving toward observer

shorter \( \lambda \)

(blue shift)

Measurements of doppler shift of stars + galaxies

were used to show universe is expanding

\[ \rightarrow \text{Hubble constant} \quad (\text{further objects are going faster}) \]
Suppose \( v_1 = -v_2 \) what happens?

and wave \( \_2 \) inverted

\[ \rightarrow \]

held fixed

\[ \rightarrow \]

Redocked wave is inverted

Newton's second law

\[ \rightarrow \]

\( y_1(x,t) \)

\[ \rightarrow \]

\( y_2(x,t) \)

\[ \rightarrow \]

\[ y_{\text{total}} = y_1(x,t) + y_2(x,t) = A \cos k(x - vt) + (-A) \cos k(x + vt) \]

so one equation with unknowns \( k, v \)

what else? boundary conditions

\[ \rightarrow \]

\( y(x=0,t) = y(x=L,t) = 0 \)

Each boundary condition gives an additional constraint that can be used to eliminate (solve) for a variable or possibly constrain possible values for that variable

used in E+M and Q.M.

Apply to this case
\[ x = 0 \text{ side} \]
\[ y(x=0, t) = 0 = A \cos k(0 - vt) + (-A) \cos k(0 + vt) \]
\[ = A \cos(-kvt) - A \cos(kvt) \quad \text{recall that cosine is a symmetric function} \]
\[ = A \cos(kvt) - A \cos(kvt) \]
\[ = 0 \]
\[ \text{so this B.C. does not yield useful information} \]

\[ \text{what about } x = L \text{ side?} \]
\[ y(x=L, t) = 0 = A \cos k(L - vt) + (-A) \cos k(L + vt) \]
\[ \text{apply } \cos(A + B) = \cos A \cos B - \sin A \sin B \]
\[ 0 = A \left[ \cos kL \cos kvt + \sin kL \sin kvt \right] \]
\[ \uparrow \text{cancel} \]
\[ - A \left[ \cos kL \cos kvt - \sin kL \sin kvt \right] \]
\[ \rightarrow 0 = 2A \sin kL \sin kvt \]
\[ \text{recall } k = \frac{2\pi}{L} \quad \text{and } \nu = \frac{\lambda}{V} \]
\[ k \nu = \frac{2\pi}{L} \frac{\lambda}{V} = \frac{2\pi}{V} = \omega \]

\[ \text{what does this mean physically?} \]

\[ \text{consider the three parts} \]
\[ \begin{align*}
\& \quad \text{(1) constant} \\
\& \quad \text{(2) } \sin kvt = \sin \omega t \quad \text{depends on } t \to -\infty \to \infty \text{ is range of } t \text{ so we can't force this term to 0} \\
\& \quad \text{this leaves (2)} \\
\& \quad \sin kL = 0 \\
\& \quad \text{so } k \text{ can only have certain values} \\
\& \quad \text{since } k = \frac{2\pi}{L} \Rightarrow \lambda \text{ can only have certain discrete values}
\end{align*} \]
recall that \( \sin(x) \) looks like

\[
\begin{align*}
\sin(x) & \quad \text{so when is } \sin(kL) = 0 \\
KL &= n\pi; \ n = 0, 1, 2, 3, \ldots
\end{align*}
\]

\[
N = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{2L}{N}
\]

So we can rewrite original equation as

\[
y(x,t) = 2A \sin\left(\frac{n\pi x}{L}\right) \sin(w t)
\]

\[
n = 1, 2, 3, \ldots
\]

\[
\begin{align*}
\text{max deflection when} & \\
\frac{\pi x}{L} &= \frac{\pi}{2} \Rightarrow x = \frac{L}{2}
\end{align*}
\]

\[
\begin{align*}
\text{max deflection when} & \\
\frac{2\pi x}{L} &= \frac{\pi}{2} \quad \text{and} \quad \frac{3\pi}{2} & \Rightarrow x = \frac{L}{4} & \quad x = \frac{3L}{4}
\end{align*}
\]

So \( n = \# \) of antinodes

DEMO WITH ROPES
Another example of interference arises when the path lengths are different. Used in a demonstration (the first) that light behaves like a wave (Newton's libid particle interpretation).

**Young's Double Slit Experiment** 1801

**Key Feature**

Consider two paths 1 and 2. They are not the same length. Path length difference is \( \Delta L \); \( \sin \theta = \frac{\Delta L}{a} \); \( \Delta L = a \sin \theta \)

So if \( \Delta L = n \lambda \rightarrow \text{constructive interference} \) (bright spots on screen)

\[ n \lambda = a \sin \theta \]

\[ \sin \theta = \frac{n \lambda}{a} \]

So given \( a, \lambda \) can predict where bright spots should appear.

Or can simplify this as

\[ \sin \theta = \tan \theta = \frac{y}{D} = \frac{n \lambda}{a} \Rightarrow y = \frac{D}{a} \times n \lambda \]

\( n = 1, 2, \ldots \) bright spots

Many ways to demonstrate this. "Ace high Demo!"
double slit experiment using precision slits

\[ \Rightarrow \text{don't see a simple pattern, Why?} \]

hint there are two effects!

interference

Effect 1: between slits

\[ y_{\text{min}} \sim \frac{D \pi \lambda}{s} \]

which effect causes which patterns?

hint recall \( y_{\text{max}} \sim \frac{D \pi \lambda}{a} \)

which has larger modulation (larger distance between bright spots?)
same effect of constructive + destructive interference in double slit case also happens for single slit.

\[ y = n \lambda \frac{D}{a} \]

now modulation for double slit is much faster (closer) than single slit. But note that \( S \ll a \), so would expect this if you work this out.

\[ Y_n = n \lambda \frac{D}{a} \quad \text{max, double slit} \]

\[ Y_n = n \lambda \frac{D}{S} \quad \text{min, single slit} \]

For \( a = 0.25 \text{mm} \) \( S = 0.08 \text{mm} \)

\( S \sim \frac{1}{3} a \) so fine pattern of bright spots arising from double slit interference, modulated about every 3 by the single slit diffraction.
Derivation of single slit diffraction

\[ \Delta L = \frac{s}{2} \sin \theta \]

\[ \Delta L = \frac{\lambda}{2} \quad \text{DESTRUCTIVE INTERFERENCE} \]

\[ \frac{s}{2} \sin \theta = \frac{\lambda}{2} \]

But could divide again

\[ \Delta L = \frac{s}{4} \sin \theta \Rightarrow \frac{\lambda}{2} \]

If we generalize

\[ \frac{s}{2n} \sin \theta = \frac{\lambda}{2} \]

\[ \sin \theta = \frac{n \lambda}{s} \quad \sin \theta = \frac{y}{D} \]

\[ y = \frac{n \lambda D}{s} \quad \text{DESTRUCTIVE INTERFERENCE SINGLE SLIT} \]