Download the Mathematica initialization file by either linking to
http://inside.mines.edu/~meberhar/classes/down_loads/f10_351_test1.nb
or going to the class website and clicking on Test 1

Read the instructions at the top of the downloaded Mathematica notebook before continuing.

Warm up (this question refers to the standard particle in a box problem)
1) arbPsi is a linear combination of particle-in-a-box wavefunctions, where the length of the box is 1. Find the first three coefficients.

The last 3 digits of your CWID in reverse order and divided by 10, i.e. if your CWID ends in 347 then your answer will be 7/10, 2/5, 3/10

Now for the harder questions (to work the remainder of the questions you will use the downloaded wavefunctions ho[n])
2) The wavefunctions, ho[n], are those corresponding to a particle moving in a potential of the form x^2/2. Graph the potential and clearly label the axes.

3) Write the Hamiltonian operator for Problem 2.

$$\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{x^2}{2}$$

4) Write the Schrödinger equation for Problem 2.

$$\frac{-\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + \frac{x^2}{2} \psi = E \psi$$
5) The wavefunctions found by solving the Schrödinger equation of Problem 4 are given by the downloaded functions \( h_0[n] \), where \( n = 1, 2, 3, 4 \ldots \). Draw rough representations of \( h_0[1] \) and \( h_0[2] \) with \( x \) between -5 and 5.

6) Draw rough representations of the probability density of \( h_0[1] \) and \( h_0[2] \) with \( x \) between -5 and 5. 

7) Find the average position of a particle with wavefunction \( h_0[3] \).

0

8) Find the average of the position squared for a particle with wavefunction \( h_0[3] \).

Credit will be given for 0 or for 2.5

9) Write down the formula to find the variance in the position of a particle with wavefunction \( h_0[n] \).

\[
\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 \quad \text{or} \quad | \langle x \rangle^2 - \langle x^2 \rangle | \quad \text{or} \quad
s[h_0[n], x^2 h_0[n]] - s[h_0[n], x h_0[n]]^2
\]
10) Find the total energy of a particle with wavefunction $\psi[2]$. (This should be a number.)

\[ s[\psi[2], \psi[2]] = \langle \psi[2] | H \psi[2] \rangle = 1.5 \]

11) **Hard question:** The total energy of a particle with wavefunction $\psi[3]$ is 5/2 (in the energy units we are using). Find the amplitude of the oscillations for a classical particle with this energy moving in a potential given by $x^2/2$. Find the probability that our quantum particle can be found in the classical region.

A pendulum is an example of a particle moving back and forth in a potential of $x^2/2$. The total energy of the pendulum is constant through its swing. At its classical turning point (where the pendulum stops and turns around) there is no kinetic energy so all of the energy is in the form of potential: Hence

\[ \frac{5}{2} = \frac{x^2}{2} \Rightarrow \text{the particle is oscillating between } x = -\sqrt{5} \text{ and } +\sqrt{5} \text{ this is the classical region:} \]

The probability of finding it in this region is given by:

\[ \int_{-\sqrt{5}}^{\sqrt{5}} \psi[3]^2 \, dx = 0.90 \]

A few people said they subtracted this number from 1 to get the probability in the non-classical region i.e. 0.10 These people got half credit.