Sensors and Image Formation

Examples
• A camera has a field of view of 90x90 degrees, an image resolution of 400x400 pixels, and that the center of the image is the optical center of the camera. A point P has 3-D coordinates (1m, 2m, 8m) in camera coordinates. Find the pixel projection of point P in the image.
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Solution: we first need to find f. With a fov of 90 degree, f=half the size of the image, or 200 pixels. Then use

\[
\begin{align*}
  x_{img} &= f \frac{X}{Z} + c_x = (200)(1m/8m) + 200 = 225 \\
  y_{img} &= f \frac{Y}{Z} + c_y = (200)(2m/8m) + 200 = 250
\end{align*}
\]
• A camera views a square lying on a plane; where the plane is parallel to the image plane
  – Show that the width of the square in the image doesn’t depend on the location
  – Find the relationship between the width of the square in the image, and the distance to the plane
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• Solution
  – Let the plane be a distance $Z$ from the camera
  – Let the sides of the square be at $X_1$ and $X_2$; $W=X_1-X_2$
  – The image projection of those sides is $x_1 = f \frac{X_1}{Z}$, $x_2 = f \frac{X_2}{Z}$
  – The image width is $w = x_1-x_2 = f \frac{W}{Z}$
A CCD sensor is 10mm x 10mm, and has 10M sensor elements. Lens focal length is 6 mm. What is the instantaneous field of view (iFov); ie the angular size of one pixel at the center?
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• Solution
  – Assuming the sensor elements are in a square grid, we have \( \sqrt{10M} = 3162 \) elements on a side of 10 mm
  – So one sensor element is \( 10\text{mm}/3162 = 0.00316 \text{ mm wide} \)
  – At the center, angle is \( \text{atan}(0.00316/6) = 5.27e-4 \text{ radian} \)
• What is the IFOV for the human eye? Assume one receptor cell on the retina is .003 mm wide, and the focal length is 17 mm

• What is width of smallest object you can see at 30m? Assume that the image of the object has to cover at least one receptor cell
• What is the IFOV for the human eye? Assume one receptor cell on the retina is 0.003 mm wide, and the focal length is 17 mm
  – Solution
    • \( \text{atan}(0.003/17) = 1.76 \times 10^{-4} \text{ radian} \)

• What is width of smallest object you can see at 30m? Assume that the image of the object has to cover at least one receptor cell
  – Solution
    • By similar triangles, \( \frac{w}{30} = \frac{0.003}{17} \Rightarrow w = 0.0053 \text{ m} \)