1. Particles are a major component of air pollution in many areas. It is of interest to study the sizes of contaminating particles. Let $X$ represent the diameter, in $\mu$m, of a randomly chosen particle. Assume that in a certain area, the probability density function of $X$ is inversely proportional to the volume of the particle, that is, assume that

$$f(x) = \frac{c}{x^3}I_{(1,\infty)}$$

where $c$ is a constant.

(a) Find the value of $c$ so that $f(x)$ is a probability density function.
(b) Find the mean particle diameter.
(c) Find the cumulative distribution function of the particle diameter.
(d) Find the median particle diameter.
(e) The term PM$_{10}$ refers to particles 10 $\mu$m or less in diameter. What proportion of the contaminating particles are PM$_{10}$?
(f) The term PM$_{2.5}$ refers to particles 2.5 $\mu$m or less in diameter. What proportion of the contaminating particles are PM$_{2.5}$?
(g) What proportion of the PM$_{10}$ particles are PM$_{2.5}$?

2. Bottles are labeled as containing 12 ounces of liquid. The process that fills the bottles produces a volume with standard deviation 0.02 ounces and whose mean can be set through calibration. Assume the fill volumes are normally distributed. To what value should the mean be set so that no more than 1% of the bottles will be underfilled?

3. Scores on an exam are normally distributed with mean 70 and standard deviation 10.

(a) What proportion of the students scored over 90?
(b) Students whose scores are above the 85th percentile get an A on the exam. Among the A students, what proportion of them scored over 90?

4. Weights of female cats of a certain breed are normally distributed with mean 4.1 kg and standard deviation 0.6 kg.

(a) What proportion of female cats have weights between 3.7 and 4.4 kg?
(b) A certain female cat has a weight that is 0.5 standard deviations above the mean. What proportion of female cats are heavier than this one?
(c) How heavy is a female cat whose weight is on the 80th percentile?
(d) A female cat is chosen at random. What is the probability that she weighs more than 4.5 kg?
(e) Six female cats are chosen at random. What is the probability that exactly one of them weighs more than 4.5 kg?