Problems 1–3 refer to the linear model \( y_i = \beta_0 + \beta_1x_{i1} + \cdots + \beta_px_{ip} + \varepsilon_i \). Note that the model contains an intercept, i.e. \( x_{0i} = 1 \).

1. Let \( e \) be the residual vector from the fit of the linear model.
   (a) Find a vector of constants \( c \) such that \( c^Te = 0 \).
   (b) Explain why the hat matrix \( H \) is positive semidefinite, but not positive definite.

2. A linear model is fit, and it turns out that \( e = y \). Find \( \sum y_i \).

3. A linear model is fit, and it turns out that \( e = 0 \). Does this imply that \( \varepsilon = 0 \)? Explain.

4. Let \( X \) be an \( n \times p \) matrix, \( n \geq p \), of full column rank. Let \( A \) be a \( p \times p \) matrix of full column rank, and let \( Z =XA \). Consider the following two linear models:
   \[
   \begin{align*}
   y &= X\beta + \varepsilon \\
   y &= Z\beta + \varepsilon
   \end{align*}
   \]
   Show that the residuals \( e \) are the same for both models.