FE Review
Economics and Cash Flow

Andrew Pederson
Teaching Associate Professor
Economics and Business

Compound Interest Variables

- **P = present single sum of money (single cash flow).**
- **F = future single sum of money (single cash flow).**
- **A = uniform series of money (multiple cash flows).**
- **n = number of compounding periods (months, years, etc.)**
- **i = period compound interest rate,**
- **i* = investor's minimum rate of return**

Pg 16

Time Diagram of Compound Interest Variables, Figure 2-1

		A	A	A ..... A	A	
P						F
	0	1	2	3 ..... n-1	n	

Pg 16

Variable Relationships

Desired Quantity	=	Given Quantity	X	Appropriate Factor
F	=	P	X	$F/P_{i,n}$
F	=	A	X	$F/A_{i,n}$
P	=	F	X	$P/F_{i,n}$
P	=	A	X	$P/A_{i,n}$
A	=	F	X	$A/F_{i,n}$
A	=	P	X	$A/P_{i,n}$
A	=	G	X	$A/G_{i,n}$

Pg 17

Factor Symbolism

- *First, note that with the factor symbolism, there is always an alternating letter symbol approach. For example, we calculate "A" given "F" using an "A/F factor."*
- *Second, the first letter in each factor describes what is being calculated.*
- *Third, think of the "/" as representing the word "given" to better understand which factor to use when.*

Pg 17

Example 2-1 Single Payment Compound-Amount Factor Illustration

Calculate the future worth that $1,000 today will have six years from now if interest is 10% per year compounded annually.

P = $1,000 - - - F = ? i = 10% per year

0 1 2 6

Pg 18

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Pg 18

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P = $1,000 - - - ?

$F = \$1{,}000(F/P_{10\%,6}) = ?$

0 1 2 6

Pg 18

741 Economic Evaluation and Investment Decision Methods

i = 10.00%

n	F/Pi,n	P/Fi,n	F/Ai,n	A/Fi,n	A/Pi,n	P/Ai,n	A/Gi,n
1	1.1000	0.9091	1.0000	1.00000	1.10000	0.9091	n/a
2	1.2100	0.8264	2.1000	0.47619	0.57619	1.7355	0.4762
3	1.3310	0.7513	3.3100	0.30211	0.40211	2.4869	0.9366
4	1.4641	0.6830	4.6410	0.21547	0.31547	3.1699	1.3812
5	1.6105	0.6209	6.1051	0.16380	0.26380	3.7908	1.8101
6	1.7716	0.5645	7.7156	0.12961	0.22961	4.3553	2.2236
7	1.9487	0.5132	9.4872	0.10541	0.20541	4.8684	2.6216
8	2.1436	0.4665	11.4359	0.08744	0.18744	5.3349	3.0045
9	2.3579	0.4241	13.5795	0.07364	0.17364	5.7590	3.3724
10	2.5937	0.3855	15.9374	0.06275	0.16275	6.1446	3.7255
11	2.8531	0.3505	18.5312	0.05396	0.15396	6.4951	4.0641
12	3.1384	0.3186	21.3843	0.04676	0.14676	6.8137	4.3884
13	3.4523	0.2897	24.5227	0.04078	0.14078	7.1034	4.6988
14	3.7975	0.2633	27.9750	0.03575	0.13575	7.3667	4.9955
15	4.1772	0.2394	31.7725	0.03147	0.13147	7.6061	5.2789
16	4.5950	0.2176	35.9497	0.02782	0.12782	7.8237	5.5493
17	5.0545	0.1978	40.5447	0.02466	0.12466	8.0216	5.8071
18	5.5599	0.1799	45.5992	0.02193	0.12193	8.2014	6.0526
19	6.1159	0.1635	51.1591	0.01955	0.11955	8.3649	6.2861
20	6.7275	0.1486	57.2750	0.01746	0.11746	8.5136	6.5081

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Example 2-1 Single Payment Compound-Amount Factor Illustration

Calculate the future worth that $1,000 today will have six years from now if interest is 10% per year compounded annually.

P = $1,000 — F = ? i = 10% per year
0 1 2 6

Solution, Future Balance, $F = P(1+i)^n = 1{,}000(1.1)^6 = \$1{,}771.56$

P = $1,000
0 1 2 6

1.7716
$F = \$1{,}000(F/P_{10\%,6}) = \$1{,}771.6$

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Single Payment Present-Worth Factor

By simply re-arranging text Equation 2-1 we can solve for the present value "P" given a future value, "F" as follows:

$P = F[1/(1+i)^n]$ 2-2

The equation $1/(1+i)^n$ is called the "single payment present worth factor," and is designated by the symbol, $P/F_{i,n}$

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Example 2-2 Single Payment Present-Worth Factor

Calculate the present value of a $1,000 payment to be received six years from now if interest is 10% per year compounded annually.

P = ? — F = $1,000
0 6

$P = F[1/(1+i)^n] = \$1{,}000(1/1.1)^6 = \$564.50$

Example 2-2 Single Payment Present-Worth Factor

Calculate the present value of a $1,000 payment to be received six years from now if interest is 10% per year compounded annually.

P = ?

0 ———————————— 6   F = $1,000

$P = F[1/(1+i)^n] = \$1,000(1/1.1)^6 = \$564.50$ or,

$P = \$1,000(\overset{?}{P/F_{10\%,6}}) = ?$

0 ———————————— 6   F = $1,000

Pg 19

741 Economic Evaluation and Investment Decision Methods

i = 10.00%

n	F/Pi,n	P/Fi,n	F/Ai,n	A/Fi,n	A/Pi,n	P/Ai,n	A/Gi,n
1	1.1000	0.9091	1.0000	1.00000	1.10000	0.9091	n/a
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20	6.7275	0.1486	57.2750	0.01746	0.11746	8.5136	6.5081

Example 2-2 Single Payment Present-Worth Factor

Calculate the present value of a $1,000 payment to be received six years from now if interest is 10% per year compounded annually.

P = ?

0 ———————————— 6   F = $1,000

$P = F[1/(1+i)^n] = \$1,000(1/1.1)^6 = \$564.50$ or,

$P = \$1,000(\overset{0.5645}{P/F_{10\%,6}}) = \$564.50$

0 ———————————— 6   F = $1,000

Pg 19

Summary of Compound Interest Formulas

Single Payment Compound-Amount Factor

$= (1+i)^n = F/P_{i,n}$

P given —————— $F = P(F/P_{i,n})$

0 . . . . n

Single Payment Present-Worth Factor

$= 1/(1+i)^n = P/F_{i,n}$

$P = F(P/F_{i,n})$ —————— F given

0 . . . . n

Uniform Series Compound-Amount Factor

$= [(1+i)^n - 1]/i = F/A_{i,n}$

A given . . A —————— $F = A(F/A_{i,n})$

0   1 . . . . n

Pg 23

Summary of Compound Interest Formulas

Sinking-Fund Deposit Factor

$= i / [(1+i)^n - 1] = A/F_{i,n}$

	$A = F(A/F_{i,n})$ . . . A	F given
0	1 . . . . n	

Capital-Recovery Factor

$= i\,(1+i)^n / [\,(1+i)^n - 1\,] = A/P_{i,n}$

P given	$A = P(A/P_{i,n})$ . . . A
0	1 . . . . n

Uniform Series Present-Worth Factor

$= [(1+i)^n - 1] / [i(1+i)^n] = P/A_{i,n}$

$P = A(P/A_{i,n})$	A given . . . A
0	1 . . . . n

Pg 23

Example 2-7 Time Value of Money Factors and Timing Considerations

A person is to receive five payments in amounts of \$300 at the end of year one, \$400 at the end of each of years two, three and four, and \$500 at the end of year five. If the person considers that places exist to invest money with equivalent risk at 9.0% annual interest, calculate the time zero lump sum settlement "P," and the end of year five lump sum settlement "F," that would be equivalent to receiving the end of period payments.

Pg 24

Example 2-7 Time Value of Money Factors and Timing Considerations

Next, determine the five equal end of year payments "A," at years one through five that would be equivalent to the stated payments.

Finally, recalculate the present value assuming the same annual payments are treated first, as beginning of period values and second, as mid-period values.

Pg 24

Example 2-7 Time Zero Lump Sum Settlement Based on End of Period Values

P = ?	-	\$300	\$400	\$400	\$400	\$500
	0	1	2	3	4	5

$$P = 300\overset{0.9174}{(P/F_{9\%,1})}+400\overset{0.8417}{(P/F_{9\%,2})}+400\overset{0.7722}{(P/F_{9\%,3})}+400\overset{0.7084}{(P/F_{9\%,4})}+500\overset{0.6499}{(P/F_{9\%,5})} = \$1{,}529$$

or,

$$P = 300\overset{0.9174}{(P/F_{9\%,1})}+400\overset{2.5313}{(P/A_{9\%,3})}\overset{0.9174}{(P/F_{9\%,1})}+500\overset{0.6499}{(P/F_{9\%,5})} = \$1{,}529$$

Pg 24

Example 2-7 Closer Look at the P/A Factor

$\overset{2.5313}{400(P/A_{9\%,3})}$

P = ?	-	P$	$400	$400	$400	-
	0	1	2	3	4	5

$\overset{2.5313}{400(P/A_{9\%,3})}$ = $1,012.52 at beginning of year 2, or end of year 1.
This is still a future value at year 1, not the desired sum at 0, so

P = ?	1,012.52					
	0	1	2	3	4	5

P = 1,012.52($P/F_{9\%,1}$) = $928.92 or, 400($P/A_{9\%,3}$)($P/F_{9\%,1}$)

Pg 24

Example 2-7 Closer Look at the P/A Factor

$\overset{2.5313}{400(P/A_{9\%,3})}$

P = ?	-	P$	$400	$400	$400	-
		0	1	2	3	

$\overset{2.5313}{400(P/A_{9\%,3})}$ = $1,012.52 at beginning of year 2, or end of year 1.
This is still a future value at year 1, not the desired sum at 0, so

P = ?	1,012.52					
	0	1	2	3	4	5

P = 1,012.52($P/F_{9\%,1}$) = $928.92 or, 400($P/A_{9\%,3}$)($P/F_{9\%,1}$)

Pg 24

Example 2-7 FV at End of Year 5 Value

-	$300	$400	$400	$400	$500	F=?
0	1	2	3	4	5	

F = 300$\overset{1.4116}{(F/P_{9\%,4})}$+400$\overset{1.2950}{(F/P_{9\%,3})}$+400$\overset{1.1881}{(F/P_{9\%,2})}$+400$\overset{1.0900}{(F/P_{9\%,1})}$+500 = $2,353

or,

F = 300$\overset{1.4116}{(F/P_{9\%,4})}$+400$\overset{3.2781}{(F/A_{9\%,3})}$$\overset{1.0900}{(F/P_{9\%,1})}$+500 = $2,353

or,

F = 1,529$\overset{1.5386}{(F/P_{9\%,5})}$ = $2,353

Pg 25

Example 2-7 Expansion of FV Calculation

-	$300	$400	$400	$400	$500	F=?
0	1	2	3	4	5	

F = 400$\overset{3.2781}{(F/A_{9\%,3})}$ = $1,311 , but it is not a year 5 future value!
This is still a present sum relative to the desired future value, so;

Example 2-7 Expansion of FV Calculation

$400(F/A_{9\%,3}) = \$1{,}311$

$(F/P_{9\%,1})$

-	\$300	\$400	\$400	\$400	\$500	F=?
0	1	2	3	4	5	

$$\overset{3.2781}{} \quad \overset{1.0900}{}$$
$$F = 400(F/A_{9\%,3})(F/P_{9\%,1}) = \$1{,}429$$

Example 2-7 FV at End of Year 5 Value

-	\$300	\$400	\$400	\$400	\$500	F=?
0	1	2	3	4	5	

1.4116 1.2950 1.1881 1.0900

$$F = 300(F/P_{9\%,4})+400(F/P_{9\%,3})+400(F/P_{9\%,2})+400(F/P_{9\%,1})+500 = \$2{,}353$$

or,

1.4116 3.2781 1.0900

$$F = 300(F/P_{9\%,4})+400(F/A_{9\%,3})(F/P_{9\%,1})+500 = \$2{,}353$$

or,

1.5386

$$F = 1{,}529(F/P_{9\%,5}) = \$2{,}353$$

Pg 25

Example 2-7 Equivalent Annual Cost

0.2571

$$A = 1{,}529(A/P_{9\%,5}) = \$393$$

or,

0.1671

$$A = 2{,}353(A/F_{9\%,5}) = \$393$$

Resulting "A" Values

-	\$393	\$393	\$393	\$393	\$393
0	1	2	3	4	5

Pg 25

2.8 Arithmetic Gradient Series

-	B	B+g	B+2g		B+(n-2)g	B+(n-1)g
0	1	2	3	....	n-1	n

$A = B \pm g(A/G_{i,n})$ Textbook Equation 2-14

Where $A/G_{i,n} = (1/i) - \{n / [(1+i)^n - 1]\}$

"n" includes the base year as the mathematical development is based on applying the gradient n-1 times.

A/G Factor developed in Appendix E, pg 788

Pg 46-47

Rule of 72

Number of periods to double your money:

$$\frac{72}{\text{Compound interest rate X 100}}$$

Interest required to double your money:

$$\frac{72}{\text{Number of years}}$$

Rule of 114

Number of periods to triple your money:

$$\frac{114}{\text{Compound interest rate X 100}}$$

Interest required to triple your money:

$$\frac{114}{\text{Number of years}}$$

Continuous Interest on Discrete Values (not covered in EBGN/CHEN 321)

$F/P_{r,n} = e^{rn}$

$P/F_{r,n} = 1/e^{rn}$

$F/A_{r,n} = (e^{rn} – 1)/(e^{r} – 1)$

$A/F_{r,n} = (e^{r} – 1)/(e^{rn} – 1)$

$P/A = (e^{rn} – 1)/(e^{r} – 1)e^{rn}$

$A/P_{r,n} = (e^{r} – 1)e^{rn}/(e^{rn} – 1)$

r = nominal interest rate compounded continuously
n = number of discrete evaluation periods
e = base of natural log (ln) = 2.7183 . . .

Overview of Continuous Interest

- **Same timing assumptions as discrete compounding**
- **You can calculate the effective rate from a continuous rate using the formula:**

  $E = e^{r} – 1$

- **The Effective rate determined on a daily basis will not be significantly different than a continuous interest rate.**

2.3 Nominal, Period and Effective Interest

Nominal = Annual

Period Interest Rate, $i = \dfrac{\text{Nominal Interest Rate}}{\text{\# Compounding Periods Per Year, m}}$

Effective Interest Rate, E = Annual Percentage Yield, APY

$E = (1+i)^m - 1$ (Textbook Eq. 2-9)

Effective Interest Rates (or APY's) generate annual interest equivalent to a nominal rate compounded "m" times throughout the year.

Re-arranging Eq. 2-9; the equivalent period interest rate "i", required to achieve a desired Effective rate is; $i = (1+E)^{1/m} - 1$

Pg 26-27

Explanation of Effective Interest Rate, E

P — 0, 1, 2 . . . . m periods/year — $F_1 = P(F/P_{i,m}) = P(1+i)^m$

P — 0, 1 period/year — $F_2 = P(F/P_{E,1}) = P(1+E)^1$

Since the initial principal, "P", is the same in each case, set $F_1 = F_2$ to make the total annual interest the same for both cases as follows:

Effective Annual Interest, $E = (1+i)^m - 1 = APY$

Pg 27

Nominal, Period and Effective Interest

Nominal Rate = 5.0% or 0.05, compounded daily.

Daily Period Interest Rate $= \dfrac{0.05}{365} = 0.000137$ or 0.0137%

Effective Rate, $E = (1+.000137)^{365} - 1 = 5.127\%$ = APY

Note: Continuous compounding on discrete sums will not be significantly different than an effective annual interest rate determined on a daily basis.

Pg 26

Variation on Period and Effective Interest

Assume a company wanted a 10% annual rate of return but was working through a cash flow model based on monthly values;

Monthly Period Interest Rate $= \dfrac{0.10}{12} = 0.008333$ or 0.8333% Incorrect

Resulting Effective Rate, $E = (1+.008333)^{12} - 1 = 0.10471$ or 10.471%

The correct period interest to effectively yield 10% (E=10%) per year is determined from Equation 2-9, re-arranged to solve for "i" as follows;

$i = (1+E)^{1/m} - 1 = (1.1)^{1/12} - 1 = 0.007974$ or 0.7974% per month

Pg 27

Chapter 3

- **Income Producing Criteria**
  - Rate of Return
  - Growth Rate of Return
  - Net Present Value
  - Benefit/Cost and Present Value Ratios
- **Service Evaluations**
  - Incremental Analysis
  - Present, Annual, or Future Cost Analysis

Pg 64

Example 3-1 Present Worth Revenue Equals Break-even Acquisition Cost

Determine the present worth of the revenue streams "I", given in alternatives "A" and "B" for minimum rates of return of 10% and 20%. This gives the initial cost that can be incurred to break-even with the 10% or 20% rate of return. Note that the cumulative revenues are the same for the "A" and "B" alternatives but the timing of the revenues is very different.

Pg 65

Example 3-1 Time Diagrams

A)	P=?	I=200	I=300	I=400	I=500
	0	1	2	3	4
B)	P=?	I=500	I=400	I=300	I=200
	0	1	2	3	4

Pg 65

Example 3-1 Solution: Case A

A)	P=?	I=200	I=300	I=400	I=500
	0	1	2	3	4

$$i=10\%,\ P_A = 200\overset{0.9091}{(P/F_{10\%,1})} + 300\overset{0.8264}{(P/F_{10\%,2})} + 400\overset{0.7513}{(P/F_{10\%,3})} + 500\overset{0.6830}{(P/F_{10\%,4})} = \$1,072$$

$$\text{or, } P_A = [200 + 100\overset{1.3812}{(A/G_{10\%,4})}]\overset{3.1699}{(P/A_{10\%,4})} = \$1,072$$

$$i=20\%,\ P_A = [200 + 100\overset{1.2742}{(A/G_{20\%,4})}]\overset{2.5887}{(P/A_{20\%,4})} = \$848$$

Pg 66

Example 3-1 Solution Case B

$$B)\quad \begin{array}{ccccc} P=? & I=500 & I=400 & I=300 & I=200 \\ \hline 0 & 1 & 2 & 3 & 4 \end{array}$$

$$i=10\%,\ P_B = 500(\overset{0.9091}{P/F_{10\%,1}}) + 400(\overset{0.8264}{P/F_{10\%,2}}) + 300(\overset{0.7513}{P/F_{10\%,3}}) + 200(\overset{0.6830}{P/F_{10\%,4}})$$

$$= \$1,147$$

$$i = 10\%,\ P_B = [500 - 100(\overset{1.3812}{A/G_{10\%,4}})](\overset{3.1699}{P/A_{10\%,4}}) = \$1,147$$

$$i = 20\%,\ P_B = [500 - 100(\overset{1.2742}{A/G_{20\%,4}})](\overset{2.5887}{P/A_{20\%,4}}) = \$965$$

Pg 66

Example 3-3 Rate of Return (ROR)

If you pay $20,000 for the asset in Example 3-2, what annual compound interest rate of return on investment dollars will be received?

$$\begin{array}{cccc} C=20,000 & I=2,000 & I=2,000\ \ldots & I=2,000 \\ \hline 0 & 1 & 2\ldots\ldots\ldots & 10 \end{array}\ L=25,000$$

The only unknown in this problem is the rate of return, "i". A present, future or annual worth equation may be used to obtain "i" by trial and error calculation.

Pg 69

Example 3-3 Solution PW Equation

$$\begin{array}{cccc} C=20,000 & I=2,000 & I=2,000\ \ldots & I=2,000 \\ \hline 0 & 1 & 2\ldots\ldots\ldots & 10 \end{array}\ L=25,000$$

Present Worth (PW) Equation at Time 0 to Determine "i"

$$20,000 = 2,000(P/A_{i,10}) + 25,000(P/F_{i,10})$$

Mathematically the equation is:

$$20,000 = 2,000[(1+i)^{10} - 1] / [i(1+i)^{10}] + 25,000[1/(1+i)^{10}]$$

Pg 70

Example 3-3 Solution

$$\text{Approx. } i = \frac{\text{Arithmetic Average Income}}{\text{Cumulative Initial Costs}} = \frac{2,000}{20,000} = 0.10$$

$$i = 10\% = 2,000(6.145) + 25,000(.3855) = 21,930$$

$$i = ? \qquad = 20,000$$

$$i = 12\% = 2,000(5.650) + 25,000(.3220) = 19,350$$

Pg 70

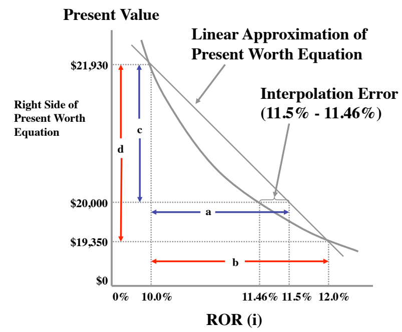
Example 3-3 Solution by Interpolation

Because there are no 11% tables in Appendix A, interpolate between the 10% and 12% values:

i = 10% + 2%[(21,930 – 20,000)/(21,930 – 19,350)] = 11.5%

This answer can also be determined graphically.

Pg 70



Pg 71

Example 3-3 Solution (Continued…)

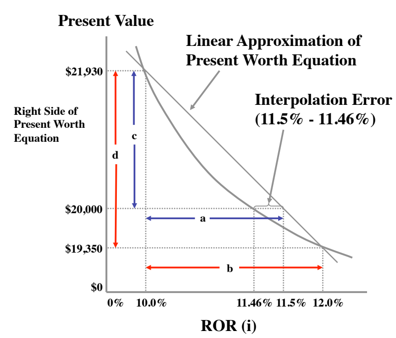
On the previous diagram, two triangles were formed. The small triangle with sides "a" and "c" is geometrically similar to the larger triangle with sides "b" and "d" since both triangles have equal angles. Therefore, the sides of the two triangles are proportional.

(a/b) = (c/d), therefore a = b(c/d) and b, c and d are known

Substituting these values gives:
a = (12%-10%)[(21,930-20,000)/(21,930-19,350)] = 1.5%

Rate of Return, i = 10% + 1.5% = 11.5%

Pg 71



Pg 71

Example 2-17 $A/P_{i,n}$ Factor Illustration

What annual end of year mortgage payments are required to pay off a $10,000 loan in five years if interest is 10% per year?

	-	A=?	A=?
P=$10,000	0	1	5

Pg 41

Example 2-17

Solution:

$$A = \$10,000(\overset{0.2638}{A/P_{10\%,5}}) = \$2,638 \text{ per year}$$

Pg 41

Example 2-17 – Loan Amortization

Yr	Beg. Balance	Payment	Interest	Principal	Ending Balance
1	$10,000	$2,638	$1,000	$1,638	$8,362
2	8,362	2,638	836	1,802	6,560
3	6,560	2,638	656	1,982	4,578
4	4,578	2,638	458	2,180	2,398
5	2,398	2,638	240	2,398	0

Pg 41

Factors to Remember in Bond Evaluations

1. *at maturity the holder will receive its "face value" as salvage or terminal value*
2. *bond cost or value will vary as market interest rates fluctuate up and down in general money markets*
3. *bond call privileges are written into most corporate or municipal bond offerings*

Pg 86

Bond Evaluation

- The value of a bond is the present worth of all future cash flows at the market interest rate.
- A bonds rate of return is the I value that makes the PW revenue equal the PW cost.

Example 3-11 New Bond Rate of Return

Calculate the bond rate of return for a new issue of $1,000 bonds with maturity date twenty years after the issuing date, if the new bond pays interest of $40 every six month period.

Pg 88

Example 3-11 Solution

-$1,000	$40	$40	$40	$1,000
0	1	2	40 semi-annual	

PW Eq: $0 = -1{,}000 + 40(P/A_{i,40}) + 1{,}000(P/F_{i,40})$

Since initial investment and maturity value are the same:

ROR, i = 40/1,000 = 4.0% per semi-annual period

The nominal ROR is 4.0% x 2 or 8.0%, which bond brokers often refer to as the bond "Yield to Maturity," for which the acronym "YTM" is utilized.

Pg 88

6 years Later (28 Semi annual periods remaining)

- Market interest rates have moved up, assume the Bond described in the prior example now sells for $800. Calculate the Bonds Yield to Maturity, Coupon Yield, and Current Yield.

Example 3-12 Solution

-$800	$40	$40	$40	$1,000
0	1	2	28	

PW Eq: $800 = 40(P/A_{i,28}) + 1{,}000(P/F_{i,28})$

i = 6% = 40(13.4062) + 1,000(0.1956) = $731.85

i = 5% = 40(14.8981) + 1,000(0.2551) = $851.02

By interpolation i = 5.43% per semi-annual period.
Yield to Maturity = Nominal ROR = 5.43% x 2 = 10.86%
Current Yield = Annual Interest / Cost = 80/800 = 10.0%
Coupon Yield = Annual Interest / Par Value = 8.0%

Pg 89

Discount Rate, i*

- Minimum Acceptable Rate of Return
- Opportunity Cost of Capital
- Financial Cost of Capital
- Weighted Average Cost of Capital
- Weighted Average Financial Cost of Capital
- Cost of Capital
- Hurdle Rate

Pg 92

Definition of i*

A compound interest measure of opportunity foregone if a different investment alternative is selected.

Pg 92

3.10 Net Present Value (NPV)

(NPV) = Present Worth Revenues or Savings @ i^*
- Present Worth Costs @ i^*

or, = Present Worth Positive Cash Flows and Negative Cash Flows Discounted @ i^*

NPV > $0 indicates a satisfactory investment
NPV = $0 is an economic breakeven
NPV < $0 is economically unsatisfactory.

Pg. 111

Example 3-21 ROR, & NPV

A five-year project requires investments of $120,000 at time zero and $70,000 at the end of year one to generate revenues of $100,000 at the end of each of years two through five. The investor's minimum rate of return is 15.0%. Calculate the Project ROR. Also, calculate the NPV. Calculate the project payback period and finally, draw an NPV Profile to show how the value of the project is impacted by the selected discount rate.

Pg 116

Example 3-21 Solution

-\$120,000	-\$70,000	\$100,000	\$100,000	\$100,000	\$100,000
0	1	2	3	4	5

Rate of Return (ROR):

PW Eq: $0 = -120{,}000 - 70{,}000(P/F_{i,1}) + 100{,}000(P/A_{i,4})(P/F_{i,1})$

@ 25% = 12,928

@ 30% = -7,212

i = 25% + 5%(12,928 / 20,140) = 28.2% > 15%, acceptable

Pg 117

Example 3-21 Solution

Net Present Value (NPV) @ i* = 15%

$$-120{,}000 - 70{,}000\underbrace{(P/F_{15\%,1})}_{0.8696} + 100{,}000\underbrace{(P/A_{15\%,4})}_{2.8500}\underbrace{(P/F_{15\%,1})}_{0.8696}$$

= $67,389 > 0, acceptable

Pg 118

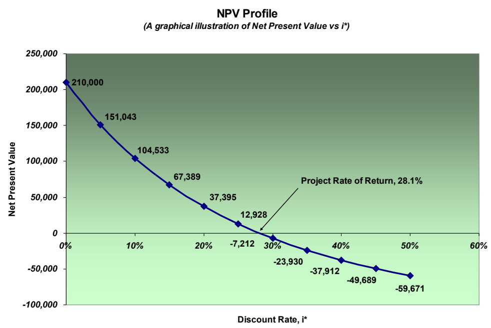
Payback

-120,000	-190,000	-90,000	10,000	90,000	190,000
0	1	2	3	4	5

2 Years + (1 Year)(90,000 / 100,000) = 2.9 Yrs

Payback is also a measure of financial risk expressed in time, it is not an overall economic measure of value added from the investments. Payback neglects time value of money.

Pg 119



Pg 120

Benefit-Cost Ratio, (B/C Ratio)

$$\text{B/C Ratio} = \frac{\text{PV Positive Cash Flow @ } i^*}{\text{PV Negative Cash Flow @} i^*}$$

B/C Ratio > 1.0 indicates satisfactory economics

B/C Ratio = 1.0 indicates break-even economics

B/C Ratio < 1.0 indicates unsatisfactory project economics

Pg. 121

Text Problem 3-20 Pg. 167-68

Problem 3-20, 21 or 22 Solutions

Year	0	1	2	3	4	5
Revenues		14,000	8,000	6,000	4,400	2,800
-Royalty Cost		-1,750	-1,000	-750	-550	-350
Net Revenue		12,250	7,000	5,250	3,850	2,450
-Operating Cost		-1,750	-1,000	-750	-500	-250
-Mine Develop.	-7,500	-2,500				
-Equipment		-6,700				
-Lease Bonus	-1,000					
Before-Tax CF	-8,500	1,300	6,000	4,500	3,350	2,200

Sol. Man. Pg 61-3

Problem 3-20, 21 or 22 Solutions

-8,500	1,300	6,000	4,500	3,350	2,200
0	1	2	3	4	5

$$\text{NPV @ 15\%} = -8{,}500 + 1{,}300\overset{0.8696}{(P/F_{15,1})} + 6{,}000\overset{0.7561}{(P/F_{15,2})} + 4{,}500\overset{0.6575}{(P/F_{15,3})}$$
$$+ 3{,}350\overset{0.5718}{(P/F_{15,4})} + 2{,}200\overset{0.4972}{(P/F_{15,5})} = +\$3{,}135 > 0\text{, accept}$$

Sol. Man. Pg 61-3

Problem 3-20, 21 or 22 Solutions

-8,500	1,300	6,000	4,500	3,350	2,200
0	1	2	3	4	5

$$\text{NPV @ 15\%} = -8{,}500 + 1{,}300\overset{0.8696}{(P/F_{15,1})} + 6{,}000\overset{0.7561}{(P/F_{15,2})} + 4{,}500\overset{0.6575}{(P/F_{15,3})}$$
$$+ 3{,}350\overset{0.5718}{(P/F_{15,4})} + 2{,}200\overset{0.4972}{(P/F_{15,5})} = +\$3{,}135 > 0\text{, accept}$$

PW Eq: $0 = -8{,}500 + 1{,}300(P/F_{i,1}) + 6{,}000(P/F_{i,2}) + 4{,}500(P/F_{i,3})$
$+ 3{,}350(P/F_{i,4}) + 2{,}200(P/F_{i,5})$

NPV @ 25% = +$777

NPV @ 30% = -$136, $i = 25\% + 5\%(777/(777+136)) = 29.3\% > i^* = 15\%$

By financial calculator, $i = ROR = 29.2\% > i^* = 15\%$, accept

Sol. Man. Pg 61-3

Problem 3-20 Breakeven Solution

X = Break-even Uniform Selling Price Per Unit:

Year	0	1	2	3	4	5
Revenues		175X	100X	75X	55X	35X
-Royalty Cost		-21.9X	-12.5X	-9.4X	-6.9X	-4.4X
Net Revenue		153.1X	87.5X	65.6X	48.1X	30.6X
-Operating Cost		-1,750	-1,000	-750	-500	-250
-Mine Develop	-7,500	-2,500				
-Mine Equip.		-6,700				
-Lease Bonus	-1,000					
Before-Tax CF	-8,500	153.1X -10,950	87.5X -1,000	65.6X -750	48.1X -500	30.6X -250

Sol. Man. Pg 61-3

Problem 3-20, 21 or 22 Breakeven Solution

$$\text{PW Eq: } 0 = -8{,}500 + (153.1X - 10{,}950)\overset{0.8696}{(P/F_{15,1})} + (87.5X - 1{,}000)\overset{0.7561}{(P/F_{15,2})}$$
$$+ (65.6X - 750)\overset{0.6575}{(P/F_{15,3})} + (48.1X - 500)\overset{0.5718}{(P/F_{15,4})} + (30.6X - 250)\overset{0.4972}{(P/F_{15,5})}$$

$0 = -19{,}681 + 285.1X$

?

Sol. Man. Pg 61-3

Problem 3-20, 21 or 22 Breakeven Solution

PW Eq: $0 = -8{,}500 + (153.1X - 10{,}950)\overset{0.8696}{(P/F_{15,1})} + (87.5X - 1{,}000)\overset{0.7561}{(P/F_{15,2})}$

$+ (65.6X - 750)\overset{0.6575}{(P/F_{15,3})} + (48.1X - 500)\overset{0.5718}{(P/F_{15,4})} + (30.6X - 250)\overset{0.4972}{(P/F_{15,5})}$

$0 = -19{,}681 + 285.1X$

Present Worth Net Cost

Sol. Man. Pg 61-3

Problem 3-20, 21 or 22 Breakeven Solution

PW Eq: $0 = -8{,}500 + (153.1X - 10{,}950)\overset{0.8696}{(P/F_{15,1})} + (87.5X - 1{,}000)\overset{0.7561}{(P/F_{15,2})}$

$+ (65.6X - 750)\overset{0.6575}{(P/F_{15,3})} + (48.1X - 500)\overset{0.5718}{(P/F_{15,4})} + (30.6X - 250)\overset{0.4972}{(P/F_{15,5})}$

$0 = -19{,}681 + 285.1X$

?

Sol. Man. Pg 61-3

Problem 3-20, 21 or 22 Breakeven Solution

PW Eq: $0 = -8{,}500 + (153.1X - 10{,}950)\overset{0.8696}{(P/F_{15,1})} + (87.5X - 1{,}000)\overset{0.7561}{(P/F_{15,2})}$

$+ (65.6X - 750)\overset{0.6575}{(P/F_{15,3})} + (48.1X - 500)\overset{0.5718}{(P/F_{15,4})} + (30.6X - 250)\overset{0.4972}{(P/F_{15,5})}$

$0 = -19{,}681 + 285.1X$

Present Worth Net Production x Selling Price, X = PW Net Revenue

Sol. Man. Pg 62

Problem 3-20, 21 or 22 Breakeven Solution

PW Eq: $0 = -8{,}500 + (153.1X - 10{,}950)\overset{0.8696}{(P/F_{15,1})} + (87.5X - 1{,}000)\overset{0.7561}{(P/F_{15,2})}$

$+ (65.6X - 750)\overset{0.6575}{(P/F_{15,3})} + (48.1X - 500)\overset{0.5718}{(P/F_{15,4})} + (30.6X - 250)\overset{0.4972}{(P/F_{15,5})}$

$0 = -19{,}681 + 285.1X$

19,681 = 285.1X or more generically, PW Cost = PW Revenue

Sol. Man. Pg 62

Problem 3-20, 21 or 22 Breakeven Solution

PW Eq: $0 = -8{,}500 + (153.1X - 10{,}950)\overset{0.8696}{(P/F_{15,1})} + (87.5X - 1{,}000)\overset{0.7561}{(P/F_{15,2})}$

$+ (65.6X - 750)\overset{0.6575}{(P/F_{15,3})} + (48.1X - 500)\overset{0.5718}{(P/F_{15,4})} + (30.6X - 250)\overset{0.4972}{(P/F_{15,5})}$

$0 = -19{,}681 + 285.1X$

$19{,}681 = 285.1X$

Present Worth Cost / Present Worth of the Net Production = \$/Unit

$19{,}681 / 285.1 = X = \$69.03$ per unit

Sol. Man. Pg 62

3.14 ROR, NPV and PVR Analysis For Service Producing Investments With Equal Lives

For rate of return, net value or ratio analysis of alternatives that provide a service, investors must make an "incremental analysis" of alternatives. Incremental analyses are made to determine if the additional up front investment(s) in the more capital-intensive alternative generates sufficient reductions in downstream operating costs (incremental savings) to justify the investment.

Pg. 141

Example 3-27 Cash Flow Solution

Incremental Setup Approach #2 – Cash Flow Sign Convention

	0	1	2	3	4	
A)	-200	-220	-240	-260	-290	50
B)	0	-300	-330	-360	-400	0
A-B)	-200	80	90	100	110	50

Pg 143

Example 3-27 Cash Flow Solution

	0	1	2	3	4	
A-B)	-200	80	90	100	110	50

$0 = -200 + 80(P/F_{i,1}) + 90(P/F_{i,2}) + 100(P/F_{i,3}) + 160(P/F_{i,4})$

Using either approach, solving for the incremental rate of return using trial and error provides the following:

@ 30% = 16

@ 40% = -19

Interpolating: ROR, i = 30% + 10%(16 / (16 + 19)) = 34.6%

34.6% > 20%, economics of automated equipment acceptable

Pg 143-144

Example 3-27 Cash Flow Solution

Incremental A-B Net Present Value @ 20%:

0.8333 0.6944 0.5787 0.4823

$0 = -200 + 80(P/F_{20,1}) + 90(P/F_{20,2}) + 100(P/F_{20,3}) + 160(P/F_{20,4})$

= +64.2 > 0, so accept automated equipment

Incremental A-B Present Value Ratio:

64.2 / 200 = 0.32 > 0, so accept automated equipment.

Pg 144-145

3.15 Cost Analysis of Services Producing Alternatives That Provide the Same Service Over the Same Period of Time

- ***It is equally valid to analyze the present, annual or future cost of providing a service for a common evaluation life.***
- ***The minimum cost analysis may be based on using the cost and revenue sign convention where costs are positive and revenues are negative. Note this is the opposite of the cash flow sign convention where revenues are positive and costs negative. Consistency in application and proper interpretation of results is really the key issue as either method is valid.***

Pg. 145

Example 3-28 Solution

A)	-200	-220	-240	-260	-290	50
	0	1	2	3	4	

B)	0	-300	-330	-360	-400
	0	1	2	3	4

Present Worth Cost (PWC_A) @ 20%

0.8333 0.6944 0.5787 0.4823

$-200 - 220(P/F_{20,1}) - 240(P/F_{20,2}) - 260(P/F_{20,3}) - 240(P/F_{20,4}) = -\$816.2$

Present Worth Cost (PWC_B) @ 20%

0.8333 0.6944 0.5787 0.4823

$-300(P/F_{20,1}) - 330(P/F_{20,2}) - 360(P/F_{20,3}) - 400(P/F_{20,4}) = -\$880.4$

Pg 146

Example 3-28 Solution

The incremental analysis presented in Example 3-27 was based on looking at the difference in the A – B costs or cash flows. If you consider the difference in the

$PWC_A - PWC_B$ or, -816.2 – (-880.4) = +64.2

This was the incremental NPV for A–B.

Pg 146

Chapter Four

- Mutually Exclusive Alternatives
  - Examples Include Develop vs Sell or, Joint Ventures, Buy vs. Explore, Financial Constraints, or Manpower Constraints.
  - When Applying Criterion, Biggest Economic Measure Not Always Best!
  - Incremental Analysis is the Key Concept!
- Non-Mutually Exclusive Alternatives
  - Ranking Exploration Prospects
  - More than one alternative may be selected
  - Objective to Maximize Cumulative Wealth!

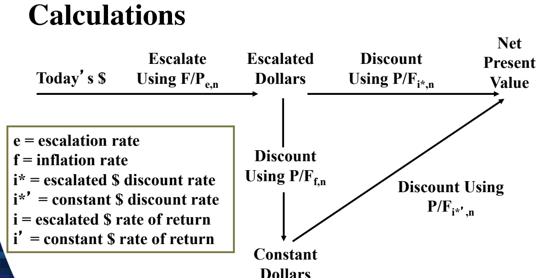
Pg 183

Inflation

- Inflation is defined as a persistent rise in the prices of a "Consumer Price Index" type basket of goods, services and commodities that is not offset by increased productivity.
  - *The Federal Reserve might define inflation as the result of too many dollars chasing too few goods.*
    - *Core Measure based on the Personal Consumption Expenditure "PCE" Index*
- Deflation refers to an overall decline in the prices for a similar basket of goods and services.

Pg 276

Equivalent Escalated Dollar and Constant Dollar Present Value Calculations



Pg 288

Section 5.3 Summary, pg. 306

Variables related to escalated and constant dollar calculations;

e = parameter escalation rate(s)
f = annual inflation rate
i* = escalated $ discount rate
i*' = constant $ discount rate
i = escalated dollar rate of return or period interest rate
i' = constant dollar rate of return or period interest rate
Eq 5-1: $(1+i) = (1+f)(1+i')$
Or, rearranged: $i' = \{(1+i) / (1+f)\} - 1$
Common Approximation: $i' = i - f$

6.5 Expected Value Analysis

- ***Expected value*** *is defined as the difference between expected profits and expected costs.*
  - ***Expected profit*** *is the probability of receiving a certain profit times the profit.*
  - ***Expected cost*** *is the probability that a certain cost will be incurred times the cost.*
- **A positive expected value is necessary, but not always a sufficient condition for an economically satisfactory investment in light of the perceived uncertainty and financial risk.**

Pg 327

Example 6-4 Expected Value Analysis of a Gambling Game

A wheel of fortune in a gambling casino has 54 different slots in which the wheel pointer can stop. 4 of the 54 slots contain the number 9. For $1 bet on hitting a 9, the gambler wins $10 plus the return of the $1 bet if he or she succeeds.

What is the expected value of this gambling game?

What is the meaning of the expected value result?

Pg 327

Example 6-4 Solution

Probability of Success = 4/54

Probability of Failure = 50/54

Expected Value = Expected Profit – Expected Cost

= (4/54)($10) – (50/54)($1) = – $0.185

Pg 328

Straight Line Depreciation (Financial)
Not Covered in EBGN/CHEN321

$$\frac{\text{(Cost Basis – Salvage Value)}}{\text{Depreciation Life}} = \text{Yearly Depreciation}$$

Table 7-3 MACRS Depreciation Rates

Year	3-Year	5-Year	7-Year	10-Year	15-Year	20-Year
	The MACRS Depreciation Rate is:					
1	.3333	.2000	.1429	.1000	.0500	.03750
2	.4445	.3200	.2449	.1800	.0950	.07219
3	.1481	.1920	.1749	.1440	.0855	.06677
4	.0741	.1152	.1249	.1152	.0770	.06177
5		.1152	.0893	.0922	.0693	.05713
6		.0576	.0892	.0737	.0623	.05285
7			.0893	.0655	.0590	.04522
8			.0446	.0655	.0590	.04522
9				.0656	.0591	.04462
10				.0655	.0590	.04461
Etc . . .						

Pg 382

Example 7-7
Depreciation Using Table 7-3

Year	7-Yr Life Rate	Initial Basis	Depreciation
1	0.1429	$100,000	$14,290
2	0.2449	$100,000	$24,490
3	0.1749	$100,000	$17,490
4	0.1249	$100,000	$12,490
5	0.0893	$100,000	$8,930
6	0.0892	$100,000	$8,920
7	0.0893	$100,000	$8,930
8	0.0446	$100,000	$4,460
Total	1.0000		$100,000

Pg 383

Book Value

Book value is the original cost basis minus the total depreciation taken.

Note: When using straight line depreciation for the FE you can not add up the remaining depreciation to calculate the book value.

Book Value

Calculate the book value of the asset in Example 7-7 after four years of MACRS depreciation?

Year	7-Yr Life Rate	Initial Basis	Depreciation
1	0.1429	$100,000	$14,290
2	0.2449	$100,000	$24,490
3	0.1749	$100,000	$17,490
4	0.1249	$100,000	$12,490
		Total Depreciation	68,760

Book Value = 100,000 – 68,760 = 31,240

Capitalized Cost
Not Covered in EBGN/CHEN321

The present worth of the costs for a project with an infinite life is known as a capitalized cost. It is the amount of money at time period zero needed to perpetually support the project .

Capitalized Cost = P = $\frac{A}{i}$

Good Luck!

If you have any questions please stop by my office and I'd be happy to answer!

Andy Pederson
Engineering Hall #125
apederso@mines.edu
apederson@me.com
Cell: (253) 320-1485