General Relativity HW 3 Quiz Name KEY

You are welcome to try both problems below, but you will only receive credit for the most correct problem.

1. (*10pts*) Given $T_{\mu\upsilon} = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ in a space with metric $g_{\mu\upsilon} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$, determine the components of $T^{[\mu\upsilon]}$.

We first need $T^{\mu\nu} = g^{\mu\alpha} T_{\alpha\lambda} g^{\lambda\nu}$ where $g^{\mu\nu} = (g_{\mu\nu})^{-1} = \begin{pmatrix} -1 & & \\ & 2 & \\ & & 2 \end{pmatrix}$

$$= \begin{pmatrix} -1 & & \\ & 2 & \\ & & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ -2 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & & \\ & 2 & \\ & & 2 \end{pmatrix} = \begin{pmatrix} -1 & -2 & 1 \\ -4 & 2 & 2 \\ -2 & 2 & 2 \end{pmatrix} \begin{pmatrix} -1 & & \\ & 2 & \\ & & 2 \end{pmatrix} = \begin{pmatrix} 1 & -4 & 2 \\ 4 & 4 & 4 \\ 2 & 4 & 4 \end{pmatrix}$$

Then $T^{[\mu\nu]} = \frac{1}{2}(T^{\mu\nu} - T^{\nu\mu}) = \frac{1}{2}\left[\begin{pmatrix} 1 & -4 & 2 \\ 4 & 4 & 4 \\ 2 & 4 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 4 & 2 \\ -4 & 4 & 4 \\ 2 & 4 & 4 \end{pmatrix}\right] = \begin{pmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

2. (*10pts*) Given $F_{\mu\upsilon} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$ show that $\partial_\mu F^{\nu\mu} = J^\nu$ where $J^\nu = (\rho, J^x, J^y, J^z)$ implies that $\vec{\nabla} \cdot \vec{E} = \rho$.

Since $J^0 = \rho$ we need the $\nu = 0$ term of $\partial_\mu F^{\nu\mu} = J^\nu$.

$$\partial_\mu F^{0\mu} = \partial_0 F^{00} + \partial_1 F^{01} + \partial_2 F^{02} + \partial_3 F^{03} = \rho$$

For $F^{\mu\nu} = \eta^{\mu\alpha} F_{\alpha\lambda} \eta^{\lambda\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

Then: $\partial_\mu F^{0\mu} = \rho \Rightarrow \frac{\partial 0}{\partial t} + \frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z = \rho \Rightarrow \vec{\nabla} \cdot \vec{E} = \rho$