# General Relativity Midterm Exam   Name_____

1. Start in $\mathbb{M}^2$, i.e. 2D Minkowski spacetime, with coordinates $\{t, x\}$. Consider an incredibly boring lifetime wherein a person is born and then dies without ever changing their spatial position.
   a) On the first spacetime diagram provided below, pick and label two events that could correspond to their birth and death, and then indicate what region of the spacetime diagram could influence their life, i.e. what events are causally connected to the events of their life?
   b) On the second spacetime diagram, using the same two events of birth and death, indicate what region of events they might be able to "see" in their lifetime.

   a)

   b)
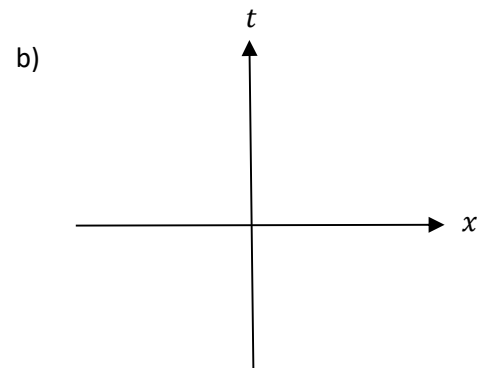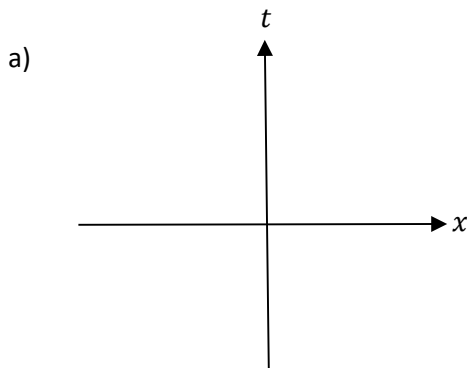
   c) How many independent transformations will leave the metric invariant?
2. Now consider the coordinate changes $u = e^t$ and $v = 2x$.
   a) Given that $t, x \in (-\infty, \infty)$, what are the ranges of u and v?
   b) What is the metric for the space in the coordinates $\{u, v\}$? Hint: There is more than one way to do this.
   c) If a dual-vector has components $A_\mu = (1,1)$ in the $\{u, v\}$ coordinate system, then what are the components of the corresponding vector A^μ?
   d) What are the nonzero Christoffel connection coefficients for this space in the $\{u, v\}$ coordinate system?
   e) Is the spacetime described by $\{u, v\}$ flat? How do you know? Hint: There is more than one way to do this.
   f) How many independent Killing vectors does this spacetime have? Explain. Hint: There is more than one way to do this.
   g) Is the path $\{u(\lambda), v(\lambda)\} = \{\lambda, \lambda\}$ a geodesic? Prove your answer.
3. Now consider the same set of points $\{u, v\}$, but now with metric $ds^2 = -vdu^2 + dv^2$. You can choose to do either of the following questions, but do not need to do more than one in order to get full credit for this part. You grade will be based on the question that you get most correct.
   a) (*Mathematica*) Is this space maximally symmetric? Provide evidence.
   b) (*Mathematica*) What are the components of the energy-momentum tensor that would create this geometry?
   c) (*By hand*) Calculate by hand the Christoffel connection coefficients for this metric in these coordinates and then take the covariant derivative of the vector given by $A^\mu = (u, v)$.