Mathematical Methods in Physics HW3

1. Consider the space P_2 of polynomials up to second order in t with the inner product defined by $(x, y) = \int_0^1 x(t)y(t)\,dt$. Using the standard basis $X = \{1, t, t^2\}$ as well as the orthonormal one we found in class $Y = \left\{1, \sqrt{12}\left(t - \frac{1}{2}\right), \sqrt{180}(t^2 - t + \frac{1}{6})\right\}$, evaluate the angle between the two vectors $x = 1 + 2t + 3t^2$ and $y = -2 + t^2$. Compare your results. Which method was easier?

2. Consider the vector space of even polynomials up to 4th degree armed with the inner product defined by $(x, y) = \int_0^1 x(t)y(t)\,dt$. A natural basis would be $X = \{x_0, x_2, x_4\} = \{1, t^2, t^4\}$.

a) Confirm that this basis is not orthogonal.
b) Starting with the basis vector x_0, use the Graham-Schmidt process to determine an orthonormal basis.
c) Starting this time with the basis vector x_2, use Graham-Schmidt to determine an orthonormal basis.

3. Consider the derivative operators $D = \frac{d}{dt}$ and $D^2 = \frac{d^2}{dt^2}$ acting on the vector space considered in question (2).

a) Are both of these linear operators on this space? Explain.
b) For any of these two that are linear operators, find their matrix representations with respect to the first two bases discussed in problem (2). Verify the matrix forms by acting on the vector $x = \alpha_0 + \alpha_2 t^2 + \alpha_4 t^4$ (rewritten in terms of each basis) with both the straight derivative(s) and via matrix multiplication of the components.
c) Find the transformation that carries the operators between each of the two basis sets.