Mathematical Methods in Physics HW4

1. Consider anti-self-adjoint matrices. Prove that their eigenvalues are purely imaginary. Find a couple of 2x2 examples of a matrices which are both anti-self-adjoint as well as unitary, and verify that their eigenvalues are imaginary (no real part).

2. Recall the three results for isometric matrices:

a) $UU^{\dagger} = I$
b) $(Ux, Uy) = (x, y)$ for all x and y.
c) $\|Ux\| = \|x\|$ for all x.

For which we showed that (a) ⇒ (b) ⇒ (c) ⇒ (a) in class. Show that (c) ⇒ (b) ⇒ (a) ⇒ (c) as well.

3. Consider the following matrices. Prove that each is normal. Then identify further properties of each, i.e. Hermitian, unitary, symmetric, orthogonal. Find the eigenvalues of each matrix, then find the corresponding eigenvectors. Then find all of the matrices which diagonalize these, and identify the type of matrices that they are, i.e. Hermitian, unitary, symmetric, orthogonal. And finally, using these matrices show the diagonal form of the original matrices.

a) $M = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$
b) $N = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

4. Consider the following statement: For a Hermitian matrix that is fully degenerate, i.e. all of its eigenvalues are the same, then the matrix is necessarily diagonal. If this is true, prove it. If not, find a counter example.