Mathematical Methods in Physics HW5

1. For the matrix $M = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, show that the standard construction of eigenvalues and eigenvectors agrees with the results of the extremizing $I = (x, Mx)$ subject to $(x, x) = 1$.

2. Consider the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the set of vectors y such that $[M - \lambda I]x = y$, where λ is an eigenvalue of M, has a solution.

3. Show that a linear operator whose combination with its adjoint, i.e. $A^{\dagger}A$, gives a nonzero scalar factor times the identity, also enjoys the property that eigenvectors associated with distinct eigenvalues will always be orthogonal. And while your at it, go ahead and address the question I asked in my notes but did not mention in class, that is, what about anti-isometric matrices? Are they normal?

4. Consider the matrix $A_0 = \begin{pmatrix} 3 & 0 & i\sqrt{8} \\ 0 & 2 & 0 \\ -i\sqrt{8} & 0 & 1 \end{pmatrix}$. Now consider a perturbation given by

$A = \begin{pmatrix} 3 & 0 & i\sqrt{8} \\ 0 & 2 & 0 \\ -i\sqrt{8} & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. For the new matrix, determine the eigenvalues up to third order in ϵ and the eigenvectors to first order in ϵ using nondegenerate perturbation theory.

5. Check that your results from problem (3) for the eigenvalues agree with what you would obtain by directly determining the eigenvalues as an expansion in ϵ.