Mathematical Methods in Physics HW8

1. Prove that $e^{z_1}e^{z_2} = e^{z_1+z_2}$.

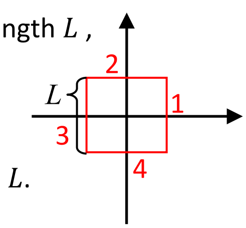
2. Consider the following functions with $z = x + iy$, and determine where, if anywhere, they are differentiable and/or analytic.

a) $w(z) = \frac{x^6-y^4+i(x^5+x^3y^3+x^2y^2+y^5)}{x^3+y^2}$
b) $w(z) = \frac{x+iy}{x+iy-1}$
c) $w(z) = \frac{|z|^2-z^*}{z^*}$

3. Consider $u(x,y) = ax^3 + bx^2y + cxy^2 + dy^3$.

a) Determine the values of a, b, c, d such that the function is harmonic.
b) Find the harmonic conjugate of $u(x,y)$.
c) Find an analytic function $w(z) = u(x,y) + iv(x,y)$ where $z = x + iy$.

4. Consider integrating the function $w(z) = z^2$ around a contour that is a square of side length L, centered around the origin with sides parallel to the real and imaginary axes. That is:



a) Show explicitly that the result is independent of the length L.
b) Use the Cauchy integral formula to find the value of $w(d)$ where d is real and $d < L$.