Math Methods HW 1 Quiz Name____________________

You can try both problems below, but you will only receive credit for the most correct solution.

1. a. (*5pts*) Consider the set $\{-1,0,1\}$ with the operation of addition. Besides the usual addition rules, what extra conditions must be imposed in order this to form a group?

Comparing to the chart for any 3 element group $\begin{matrix} * & I & A & B \\ I & I & A & B \\ A & A & B & I \\ B & B & I & A \end{matrix}$ we see that $I = 0$ and obviously $-1+1=0, -1+0=-1, 1+0=1$. Less obvious is that $-1+-1=1$ and $1+1=-1$. That is

$$\begin{matrix} + & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 0 & 1 \end{matrix}$$

b. (*5pts*) Consider the set of all x, y, z such that $x + y + z = 1$. Do these form a vector space with ordinary + and · playing their usual roles?

First of all does $\{V, +\}$ form a form an abelian group with identity $(0,0,0)$? No, since $(0,0,0)$ does not satisfy the defining condition that $x + y + z = 1$. So no it does not.

Turn over for second problem!!

2 a. (*5pts*) Recall that rational numbers are those that can be expressed by a ratio of integers, i.e. $\frac{a}{b}$ and include 0. Do the rational numbers form a field? Explain.

Yes they do. With addition, rationals add to other rationals to give rationals. The identity is 0 (which is rational) and the inverse is the negative of a rational number. Also the addition is abelian and associative. If we remove 0, then the remaining rationals form an abelian group under multiplication with 1 being the identity. The multiplication of two rationals gives another rational, and the multiplicative inverse is also rational. The multiplication is abelian and associative.

b.(*5pts*) Consider two groups:

i) The set of rotations in 2D that include $\{I, R_{90}, R_{180}, R_{270}\}$

ii) the set of rotations in 3D that include $\{I, R_{x-180}, R_{y-180}, R_{z-180}\}$.

Are these two groups isomorphic? Explain.

No they are not. Forming the multiplication tables we find:

i)

	I	R 90	R180	R270
I	I	R 90	R180	R270
R 90	R 90	R 180	R 270	I
R 180	R 180	R 270	I	R 90
R270	R270	I	R 90	R 180

or

	I	A	B	C
I	I	A	B	C
A	A	B	C	I
B	B	C	I	A
C	C	I	A	B

ii)

	I	R x-180	R y-180	R z-180
I	I	R x-180	R y-180	R z-180
R x-180	R x-180	I	R z-180	R y-180
R y-180	R y-180	R z-180	I	R x-180
R z-180	R z-180	R y-180	R x-180	I

or

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	I	A
C	C	B	A	I