# Math Methods HW 2 Quiz Name____________________

**You can try both problems below, but you will only receive credit for the most correct solution. Remember, you only have to do one, so put your best into one, not half your best into both! My past history of putting the second problem on the second page is over. Now both appear on the first page.**

1. (*10pts*) Is $M = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ diagonalizable? Explain your reasoning using the tools we have developed so far. If it is, then find its diagonal form.

Let's take $M$ and find its eigenvalues and eigenvectors. Eigenvalues first:

$$\det(M - \lambda I) = det \begin{pmatrix} 2-\lambda & 2 \\ 1 & 1-\lambda \end{pmatrix} = (2-\lambda)(1-\lambda) - 2 = \lambda^2 - 2\lambda - \lambda + 2 - 2 = \lambda^2 - 3\lambda = 0$$

Which yields $\lambda = 0$ and $\lambda = 3$. Now eigenvectors:

$$Mx = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{matrix} 2a + 2b = 0 \\ a + b = 0 \end{matrix} \rightarrow a = -b \rightarrow x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$Mx = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 3\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3a \\ 3b \end{pmatrix} \rightarrow \begin{matrix} 2a + 2b = 3a \\ a + b = 3b \end{matrix} \rightarrow 2b = a \rightarrow x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

It is diagonalizable, and moreover $M_{diag} = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}$.

2. (*10pts*) For the vector space $P_3$ with an element $x = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3$ consider the linear transformation $Dt$, that multiplies by $t$ and then takes the derivative with respect to $t$. Using the basis $\{1, t, t^2, t^3\}$ find the matrix version of this transformation, and also the matrix version of $DtDt$.

First let's evaluate: $Dtx = D(\alpha_0 t + \alpha_1 t^2 + \alpha_2 t^3 + \alpha_3 t^4) = \alpha_0 + 2\alpha_1 t + 3\alpha_2 t^2 + 4\alpha_3 t^3$

So as a matrix acting on the 4-tuple we have: $\begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ 2\alpha_1 \\ 3\alpha_2 \\ 4\alpha_3 \end{pmatrix}$ so by inspection

we have $Dt = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$.

And finally $DtDt = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 16 \end{pmatrix}$