Math Methods HW 3 Quiz Name____________________

You can try both problems below, but you will only receive credit for the most correct solution.

1. Consider the space of even degree polynomials of degree less than or equal to 4. Also consider a vector given by: $x = \left(\frac{3-\sqrt{45}}{3} + \frac{24}{1225}\right) + \left(\sqrt{45} - \frac{48}{35}\right)t^2 + \frac{8}{35}t^4$ in the basis $\{1, t^2, t^4\}$, and additionally by: $x = 1 + \sqrt{45}\left(t^2 - \frac{1}{3}\right) + \frac{8}{35}\left(t^4 - \frac{6}{7}t^2 + \frac{3}{35}\right)$ after the Gram-Schmidt process to the basis $\left\{1, \frac{\sqrt{45}}{2}\left(t^2 - \frac{1}{3}\right), \frac{8}{105}\left(t^4 - \frac{6}{7}t^2 + \frac{3}{35}\right)\right\}$.

Find the magnitude of x, i.e. $\|x\|$, using either basis.

I will use the orthonormal basis here, but the answer would be the same with either.

$x = 1 + \sqrt{45}\left(t^2 - \frac{1}{3}\right) + \frac{8}{35}\left(t^4 - \frac{6}{7}t^2 + \frac{3}{35}\right) = 1\hat{v}_1 + 2\hat{v}_2 + 3\hat{v}_3$ where $\hat{v}_1, \hat{v}_2, \hat{v}_3$ are just standing in for the orthonormal basis vectors and satisfy $(\hat{v}_i, \hat{v}_j) = \delta_{ij}$.

Then $\|x\| = \sqrt{(x,x)} = \sqrt{1+4+9} = \sqrt{14}$.

2. Consider the basis $\{x_0, x_1, x_3\} = \{1, t, t^3\}$ in the space of polynomials consisting of a constant plus linear and cubic terms.

a) (*7pts*) Going in the order $0 \to 1 \to 3$, Graham-Schmidt the shit out of it. No need to evaluate the normalization factor on the third vector.

$$y_1 = \frac{x_o}{\|x_o\|} = 1$$

$$y_2 = \frac{x_1 - (y_1, x_1)y_1}{\|x_1 - (y_1, 1)y_1\|} = \frac{t - \int_0^1 t\,dt}{\left\|t - \int_0^1 t\,dt\right\|} = \frac{t - \frac{1}{2}}{\sqrt{\int_0^1 (t-\frac{1}{2})^2\,dt}} = \sqrt{12}\left(t - \frac{1}{2}\right)$$

$$y_3 = \frac{x_3 - (y_1, x_3)y_1 - (y_2, x_3)y_2}{\|x_3 - (y_1, x_3)y_1 - (y_2, x_3)y_2\|} = \frac{t^3 - \int_0^1 t^3\,dt - \sqrt{12}(t-\frac{1}{2})\int_0^1 \sqrt{12}(t-\frac{1}{2})t^3\,dt}{\left\|t^3 - \int_0^1 t^3\,dt - \sqrt{12}(t-\frac{1}{2})\int_0^1 \sqrt{12}(t-\frac{1}{2})t^3\,dt\right\|}$$

$$= \frac{t^3 - \frac{1}{4} - \frac{9}{10}(t-\frac{1}{2})}{\left\|t^3 - \frac{1}{4} - \frac{9}{10}(t-\frac{1}{2})\right\|}$$

b) (*3pts*) Is any derivative operator a linear transformation on this space? If yes, which one(s).

$D^2 = \frac{d^2}{dt^2}$ is, as well as any derivative of higher order.