Math Methods HW 8 Quiz Name___________________

You can try both problems below, but you will only receive credit for the most correct solution.

1.(*10 pts*) Consider the following function $w(z) = \frac{1}{3}x^3 + ix + y + iy + 3i - 2$ with $z = x + iy$, and determine where, if anywhere, it is differentiable and/or analytic.

First of all let's write:

$$w(z) = \frac{1}{3}x^3 + y - 2 + i(x + y + 3) \;\Rightarrow\; u(x,y) = \frac{1}{3}x^3 + y - 2,\; v(x,y) = x + y + 3$$

Now for differentiability we can check the Cauchy-Riemann conditions:

$$\left.\begin{array}{r} \frac{\partial u}{\partial x} = x^2 \\ \frac{\partial v}{\partial y} = 1 \end{array}\right\} \Rightarrow x^2 = 1$$

$$\left.\begin{array}{r} \frac{\partial v}{\partial x} = 1 \\ -\frac{\partial u}{\partial y} = -1 \end{array}\right\} \Rightarrow 1 = -1$$

The second condition is impossible, so the function is nowhere differentiable. Hence this function is nowhere analytic.

2. Consider $u(x,y) = ax^2 - by^2 - c$.

a) (*5 pts*) Determine the values of a, b, c such that the function is harmonic.

We need $u(x,y)$ to satisfy $\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$?

$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0 \Rightarrow \quad a - b = 0 \quad \Rightarrow a = b$, and c can be anything.

b) (*5 pts*) Find the analytic function $w(z) = u(x,y) + iv(x,y)$ where $z = x + iy$ and $u(x,y)$ is as given above.

We need the harmonic conjugate to $u(x,y)$.

Start with: $\frac{\partial u}{\partial x} = 2ax = \frac{\partial v}{\partial y} \quad \Rightarrow \quad v(x,y) = \int 2ax\, dy = 2axy + f(x)$

Demand: $\frac{\partial u}{\partial y} = -2by = -2ay = -\frac{\partial v}{\partial x} \quad \Rightarrow \quad \frac{\partial v}{\partial x} = 2ay + \frac{\partial f}{\partial x} = 2ay \quad \Rightarrow \quad f(x) = constant = k$

So $w(z) = a(x^2 - y^2) - c + i(2axy + k) = az - c + ik$