Particle Physics HW1

1. For the example we have been using in the lecture of the square, extend the analysis to include the transformations that would arise if the square was embedded in three dimensions. Note: You are not generalizing to a cube, but rather just a square in 3D. The example in lecture only considered rotations in the plane. If we add the third dimension, then we get four more rotations that leave the overall shape unchanged, but relabel the corners. For the group including these transformations (as well as the 90^o rotations in the plane we already discussed) construct the relevant matrix operators and show how they act on the three representations r_1, r_2 and r_3 discussed in class. Construct the multiplication table for this enlarged group. Is it abelian?
2. For the square example in class (embedded in 2D) it might be intuitive that a faithful two-dimensional representation should exist, i.e. using two component column matrices instead of four component ones as we did in class. Explicitly construct such a representation along with the matrices that correspond to the four transformations. You do not need to consider the "extended" case from problem 1 unless you really want to. If it wasn't intuitive to you that a 2D representation should exist, figure out why it should.
3. Okay, now construct a faithful one-dimensional representation of the 2D "square group" and the corresponding "matrix" transformations.
4. The rotations that we speak of come in two types: active vs. passive. For active rotations, we use a fixed coordinate system and then actually rotate the vectors. While for a passive rotation, we keep the vectors fixed and simply rotate the coordinates. The latter of these is more useful in physics, since coordinates are our choosing, and they are used to describe real fixed vectors, i.e. your velocity is a physically fixed vector, and the coordinates we use to describe it are arbitrary. So consider the 2D "rotation" matrix $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. Determine whether this is an active or a passive transformation. Hint: To do so, I would suggest you select a starting vector and consider what would happen if you rotate it by 90^o both actively and passively. Write down the vector in terms of its components, then act on it with the matrix. Which one does it give you?
5. Show that SO(1,2) has three free continuous parameters using the defining relations for the group as real three-component vectors and metric $g = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.