Particle Physics HW7

1. For the gauged Higgs Lagrangian given in class, insert the expansion in small fluctuations about the nontrivial background $\phi_1 = \frac{1}{\sqrt{2}}\frac{\mu}{\lambda}, \phi_2 = \frac{1}{\sqrt{2}}\frac{\mu}{\lambda}, A_\mu = 0$ to obtain the Lagrangian for the fluctuations (η, β, A_μ). In particular, identify which terms in the final expression would correspond to kinetic, mass and more general interaction terms. You should see that the two scalar fluctuations both have mass in this case. However you should also see that there is a new interaction term which is only quadratic in the scalar fluctuations. The other interaction terms are cubic or higher. This interaction term is telling us that one scalar particle can freely interchange with the other, which means they are not physically distinguishable states. To get the physically distinguishable states, we should work with a linear combination for which this interaction is absent, e.g. the case done in lecture. **This will be a lengthy but straightforward calculation.**
2. Consider a Lagrangian for three real scalar fields $(\varphi_1, \varphi_2, \varphi_3)$ of the form:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{1}{2}\partial_\mu\varphi_3\partial^\mu\varphi_3 - \frac{\mu^2}{2}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2) + \frac{\lambda^2}{4}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2)^2$$

   If this three component scalar field is the Higgs field, determine the number and mass of the Higgs boson(s) as well as the number of Goldstone boson(s).

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