Particle Physics HW9

1. Consider the process $e \to e + \mu + \mu^+$. Compute the expression for $\langle |M|^2 \rangle$ to lowest order leaving it in terms of traces. This should be very analogous to what we did and where we got to by the end of lecture on Thursday! The only difference is that in doing the spin sum/average, this time you will encounter an antimatter factor for which the corresponding replacement is stated in the lecture notes.
2. Consider the process $e + e^+ \to \mu + \mu^+ + \tau + \tau^+$. For this process evaluate M for any one of the lowest order diagrams (should have four vertices). You can leave this in terms of spinor sandwiches. Make sure to read step 8 of the Feynman rules for QED from lecture for this one. This problem will require you to be very careful about matching up the spacetime indices on gamma matrices from vertices and photon propagators.
3. Consider the matrix $U = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ which acts on color states according to $c \to c' = Uc$.

   When acting with U it is helpful to remember the color states $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, and you can use that $\bar{c} \to \bar{c}' = U^* \bar{c}$ with $\bar{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \bar{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \bar{g} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

   a) Verify that U is an element of $SU(3)$.
   b) Verify that the color singlet $|9\rangle$ is invariant under U.
   c) Determine the transformation of $|8\rangle$ under U.
   d) Write the result of part (c) in terms of linear combinations of the original $|i\rangle$ where $i \in \{1,8\}$.