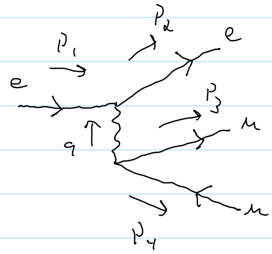
1. $e \to e + \mu + \mu^+$

Only 1 diagram at lowest (2nd) order:



$$M = \int \bar{u}(3)\, i g_e \gamma^\mu v(4)\, \bar{u}(2)\, i g_e \gamma^\nu u(1)\, \frac{-i\eta_{\mu\nu}}{q^2} (2\pi)^4 \delta^4(p_1 - p_2 + q)(2\pi)^4 \delta^4(-q - p_3 - p_4) \frac{d^4 q}{(2\pi)^4}$$

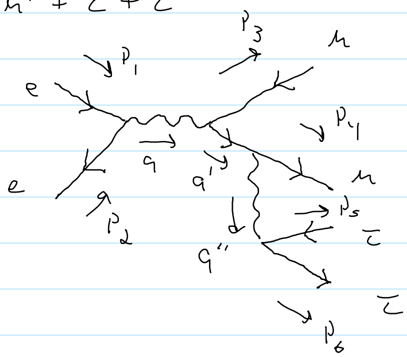
$$= -g_e^2\, \bar{u}(3)\gamma^\mu v(4)\, \bar{u}(2)\gamma^\nu u(1) \frac{\eta_{\mu\nu}}{(p_1 - p_2)^2}$$

$$|M|^2 = \frac{g_e^4}{(p_1-p_2)^4}\, \bar{u}(3)\gamma^\mu v(4)\, \bar{u}(2)\gamma^\nu u(1)\, \eta_{\mu\nu} \left[\bar{u}(3)\gamma^\lambda v(4)\, \bar{u}(2)\gamma^\beta u(1)\right]^* \eta_{\lambda\beta}$$

$$\langle |M|^2 \rangle = \underbrace{\frac{g_e^4}{2(p_1-p_2)^4}}_{\text{only 2 possible incoming spin states}} \mathrm{Tr}\left[\gamma^\mu (\not{p}_4 - m_\mu c)\bar{\gamma}^\lambda (\not{p}_3 + m_\mu c)\right] \mathrm{Tr}\left[\gamma^\nu (\not{p}_1 + m_e c)\bar{\gamma}^\beta (\not{p}_2 + m_e c)\right] \eta_{\mu\nu}\eta_{\lambda\beta}$$

2. $e + e^+ \rightarrow \mu + \mu^+ + \tau + \tau^+$

Consider :



$$M = \iiint \bar{u}(6)\, i g_e \gamma^\mu\, v(5)\, \bar{v}(2)\, i g_e \gamma^\nu\, u(1)\, \bar{u}(4)\, i g_e \gamma^\lambda\, \frac{i(\not{q}_1 + m_\mu c)}{q_1^2 - m_\mu^2 c^2}\, i g_e \gamma^\beta\, v(3)$$

$$\times \frac{-i\eta_{\mu\lambda}}{q'''^2} \times \frac{i\eta_{\nu\beta}}{q^2}\, (2\pi)^4 \delta^4(p_1 + p_2 - q)$$

$$(2\pi)^4 \delta^4(q - p_3 - q')\, (2\pi)^4 \delta^4(q' - p_4 - q'')\, (2\pi)^4 \delta^4(q'' \quad p_5 - p_6)$$

$$\frac{d^4 q}{(2\pi)^4}\, \frac{d^4 q'}{(2\pi)^4}\, \frac{d^4 q''}{(2\pi)^4}$$

3. $U = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

a) $U^{+}U = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ✓ $\quad \det U = 0(0) - 1(0-1) + 0(0) = +1$ ✓

b) Consider how U acts on $\underset{\begin{pmatrix}1\\0\\0\end{pmatrix}}{r}, \underset{\begin{pmatrix}0\\1\\0\end{pmatrix}}{b}, \underset{\begin{pmatrix}0\\0\\1\end{pmatrix}}{g}$: $\quad Ur = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} = g \quad Ub = \begin{pmatrix}1\\0\\0\end{pmatrix} = r \quad Ug = \begin{pmatrix}0\\1\\0\end{pmatrix} = b$

Similarly $\bar{U} = U^{+}$ acts on $\underset{\begin{pmatrix}1\\0\\0\end{pmatrix}}{\bar{r}}, \underset{\begin{pmatrix}0\\1\\0\end{pmatrix}}{\bar{b}}, \underset{\begin{pmatrix}0\\0\\1\end{pmatrix}}{\bar{g}}$: $\quad \bar{U}\bar{r} = \bar{g} \quad \bar{U}\bar{b} = \bar{r} \quad \bar{U}\bar{g} = \bar{b}$

Thus on $|9\rangle = \frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g}) \rightarrow \frac{1}{\sqrt{3}}(g\bar{g} + r\bar{r} + b\bar{b}) = |9\rangle$

c) On $|8\rangle = \frac{1}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g}) \rightarrow \frac{1}{\sqrt{6}}(g\bar{g} + r\bar{r} - 2b\bar{b}) = |8\rangle'$

d) $|8\rangle' = \frac{A}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g}) + \frac{B}{\sqrt{2}}(r\bar{r} - b\bar{b}) = \frac{A + \sqrt{3}B}{\sqrt{6}} r\bar{r} + \frac{A - \sqrt{3}B}{\sqrt{6}} b\bar{b} - \frac{2A}{\sqrt{6}} g\bar{g}$

So we need $\quad A + \sqrt{3}B = 1, \quad \underbrace{A - \sqrt{3}B = -2}, \quad \underbrace{2A = -1}$

$$-\frac{1}{2} + \frac{\sqrt{3}\,3}{2\sqrt{3}} = 1 \checkmark \Leftarrow -\frac{1}{2} - \sqrt{3}B = -2 \Leftarrow A = -\frac{1}{2}$$

$$B = \frac{3}{2\sqrt{3}}$$

So $|8\rangle' = -\frac{1}{2}|8\rangle + \frac{3}{2\sqrt{3}}|3\rangle$