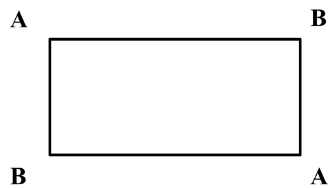
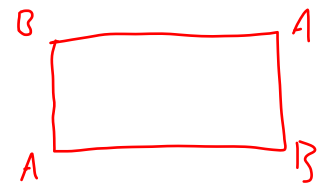
Particle Physics HW 1 Quiz   Name KEY

You can try both problems below, but you will only receive credit for the most correct solution.

1. (*6pts*) Consider the set of transformations in 3D on a rectangle which carries corners into corners based on the representation pictured. Draw and label the full set of configurations, select a set of basis vectors and construct the corresponding group transformations as matrices.



There is only one nontrivial transformation



$v_1 = (1)$ $v_2 = (-1)$

$G = \{1, -1\}$ Note: $1 v_1 = v_1$ $-1 v_1 = v_2$

$1 v_2 = v_2$ $-1 v_2 = v_1$

You could do this with larger vectors, but there is no need. For example you could use $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ then $G = \left\{ \begin{pmatrix}1&0\\0&1\end{pmatrix}, \begin{pmatrix}0&1\\1&0\end{pmatrix} \right\}$

(*4pts*) Construct the multiplication table for this group. Is it abelian or non-abelian?

	1	-1
1	1	-1
-1	-1	1

Yes it is abelian!

Turn over for second problem!!

2. (*10pts*) Consider the group $U(1,1)$ which is comprised of 2x2 matrices with complex elements that satisfy $A^\dagger g A = g$ where $g = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Determine the number of continuous free parameters in this group.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are complex, then we have 8 parameters to start.

The applying that $A^\dagger g A = g$ we find:

$$\begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}\begin{pmatrix} -a & -b \\ c & d \end{pmatrix} = \begin{pmatrix} -aa^* + cc^* & -a^*b + c^*d \\ -b^*a + d^*c & -bb^* + dd^* \end{pmatrix}$$

This should equal $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ giving:

$$\left.\begin{array}{ll} -aa^* + cc^* = -1 & \text{1 real eqn.} \\ \left.\begin{array}{l} -a^*b + c^*d = 0 \\ -b^*a + d^*c = 0 \end{array}\right] & \text{Same but complex so 2 real eqns.} \\ -bb^* + dd^* = 1 & \text{1 real eqn.} \end{array}\right\} \text{4 real eqns.}$$

In the end we have $8 - 4 = 4$ free parameters in $U(1,1)$.