

Particle Physics HW 4 Quiz

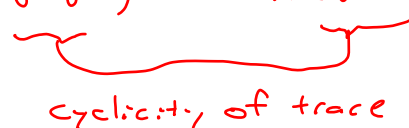
Name KEY

You can try both problems below, but you will only receive credit for the most correct solution.

1. You may use any of the results that you showed in the homework without proof.
 - a. (5pts) Evaluate $Tr(\sigma^{\mu\nu})$.
 - b. (5pts) Show that $Tr(\gamma^5\gamma^\nu) = 0$.

$$\begin{aligned}
 a) \quad Tr(\sigma^{\mu\nu}) &= Tr\left(\frac{i}{4}[\gamma^\mu, \gamma^\nu]\right) \\
 &= \frac{i}{4} \left(Tr(\gamma^\mu\gamma^\nu) - Tr(\gamma^\nu\gamma^\mu) \right) \\
 &= \frac{i}{4} \left(4\eta^{\mu\nu} - 4\eta^{\nu\mu} \right) \quad \text{from HW} \\
 &= \frac{i}{4} \left(4\eta^{\mu\nu} - 4\eta^{\mu\nu} \right) \quad \text{since } \eta^{\mu\nu} = \eta^{\nu\mu} \\
 &= 0 \quad \text{from HW}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad Tr(\gamma^5\gamma^\nu) &= Tr(-\gamma^\nu\gamma^5) = -Tr(\gamma^\nu\gamma^5) = -Tr(\gamma^5\gamma^\nu) \\
 &= 0 \quad \text{since } X = -X
 \end{aligned}$$



Turn over for second problem!!

2) Consider the Lagrangian density

$$\mathcal{L} = i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - q \bar{\psi} \gamma^\mu \psi A_\mu$$

a. (5pts) Find the field equation for variation with respect to ψ .

b. (5pts) Find the field equation for variation with respect to A_μ . You may use any results from your homework.

$$a) \frac{\partial \mathcal{L}}{\partial \psi} = -mc^2 \bar{\psi} - q \bar{\psi} \gamma^\mu A_\mu$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = i\hbar c \bar{\psi} \gamma^\mu$$

$$\text{Then: } \frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \right) = 0 = -mc^2 \bar{\psi} - q \bar{\psi} \gamma^\mu A_\mu - i\hbar c \gamma^\mu \partial_\mu \bar{\psi}$$

$$b) \frac{\partial \mathcal{L}}{\partial A_\mu} = -q \bar{\psi} \gamma^\mu \psi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} = -\frac{1}{4\pi} F^{\mu\nu} \quad \text{from HW}$$

$$\text{Then: } \frac{\partial \mathcal{L}}{\partial A_\mu} - \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} \right) = 0 = -\frac{1}{4\pi} \partial_\nu F^{\mu\nu} - q \bar{\psi} \gamma^\mu \psi$$

$$\text{or } \frac{1}{4\pi} \partial_\nu F^{\mu\nu} = q \bar{\psi} \gamma^\mu \psi$$