Particle Physics HW 4 Quiz Name KEY

You can try both problems below, but you will only receive credit for the most correct solution.

1. You may use any of the results that you showed in the homework without proof.
   a.(*5pts*) Evaluate $Tr(\sigma^{\mu\nu})$.
   b.(*5pts*) Show that $Tr(\gamma^5\gamma^\nu) = 0$.

a) $Tr(\sigma^{\mu\nu}) = Tr\left(\frac{i}{4}[\gamma^\mu, \gamma^\nu]\right)$

$= \frac{i}{4}\left(Tr(\gamma^\mu\gamma^\nu) - Tr(\gamma^\nu\gamma^\mu)\right)$

$= \frac{i}{4}\left(4\eta^{\mu\nu} - 4\eta^{\nu\mu}\right)$ from HW

$= \frac{i}{4}\left(4\eta^{\mu\nu} - 4\eta^{\nu\mu}\right)$ since $\eta^{\mu\nu} = \eta^{\nu\mu}$

$= 0$

b) $Tr(\gamma^5\gamma^\nu) = Tr(-\gamma^\nu\gamma^5) = -Tr(\gamma^\nu\gamma^5) = -Tr(\gamma^5\gamma^\nu)$ (from HW; cyclicity of trace)

$= 0$ since $X = -X$

Turn over for second problem!!

2) Consider the Lagrangian density

$$\mathcal{L} = i\hbar c\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\bar{\psi}\psi - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - q\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

a.(*5pts*) Find the field equation for variation with respect to ψ.

b.(*5pts*) Find the field equation for variation with respect to A_{μ}. You may use any results from your homework.

a) $\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = -mc^2\psi - q\psi\gamma^{\mu}A_{\mu}$

$\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\bar{\psi})} = i\hbar c\psi\gamma^{\mu}$

Then: $\frac{\partial \mathcal{L}}{\partial \bar{\psi}} - \partial_{\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\bar{\psi})}\right) = 0 = -mc^2\psi - q\psi\gamma^{\mu}A_{\mu} - i\hbar c\gamma^{\mu}\partial_{\mu}\psi$

b) $\frac{\partial \mathcal{L}}{\partial A_{\mu}} = -q\bar{\psi}\gamma^{\mu}\psi$

$\frac{\partial \mathcal{L}}{\partial(\partial_{\nu}A_{\mu})} = -\frac{1}{4\pi}F^{\mu\nu}$ from HW

Then: $\frac{\partial \mathcal{L}}{\partial A_{\mu}} - \partial_{\nu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_{\nu}A_{\mu})}\right) = 0 = -\frac{1}{4\pi}\partial_{\nu}F^{\mu\nu} - q\bar{\psi}\gamma^{\mu}\psi$

or $\frac{1}{4\pi}\partial_{\mu}F^{\mu\nu} = q\bar{\psi}\gamma^{\mu}\psi$