Particle Physics HW 6 Quiz Name KEY

You can try both problems below, but you will only receive credit for the most correct solution.

1. Consider a theory of a free Dirac fermion field that enjoys a global $U(1) \times U(1)$ invariance (note that this does NOT act on left and right separately, but both at the same time). For the following you do **not** need to show your work, but you must be very clear in notating your answers.
   a) (*3 pts*) Write down a Lagrangian for this theory, specifying the form of the transformation on the matter fields.
   b) (*3 pts*) Promote this to a local symmetry by writing down a covariant derivative and transformation rule for the gauge field(s).
   c) (*3 pts*) Write down gauge invariant kinetic term(s) for the gauge field(s). You must include the explicit form of $F_{\mu\nu}$.
   d) (*1 pt*) Draw what you need for this step.

a) $\mathcal{L} = i\hbar c\,\bar\psi \gamma^\mu \partial_\mu \psi$ where we can act w/

$\psi \to \psi' = e^{-ig_1\theta_1}\psi$

and

$\psi \to \psi' = e^{-ig_2\theta_2}\psi$

b) $\partial_\mu \to D_\mu = \partial_\mu + ig_1 A_{1\mu} + ig_2 A_{2\mu}$

where $A_{1\mu} \to A'_{1\mu} = A_{1\mu} + \partial_\mu \theta_1$

$A_{2\mu} \to A'_{2\mu} = A_{2\mu} + \partial_\mu \theta_2$

c) $-\frac{1}{16\pi} F_{1\mu\nu}F_1^{\mu\nu} - \frac{1}{16\pi} F_{2\mu\nu}F_2^{\mu\nu}$

where $F_{1\mu\nu} = \partial_\mu A_{1\nu} - \partial_\nu A_{1\mu}$

$F_{2\mu\nu} = \partial_\mu A_{2\nu} - \partial_\nu A_{2\mu}$

d) 

Turn over for second problem!!

2) An important gauge symmetry that arises in String Theory is $SO(32)$. Consider a free real scalar field that is invariant under a global $SO(32)$, that is the scalar field has 32 components. For the following you do **not** need to show your work, but you must be every clear in notating your answers. You may use that the Lie algebra of $SO(32)$ is $[g_i, g_j] = if^{ijk}g_k$.

a) (*3 pts*) Write down a Lagrangian for this theory, specifying the form of the transformation on the matter field.
b) (*3 pts*) Promote this to a local symmetry by writing down a covariant derivative and transformation rule for the gauge field(s).
c) (*3 pts*) Write down gauge invariant kinetic term(s) for the gauge field(s). You must include the explicit form of $F_{\mu\nu}$.
d) (*1 pt*) Draw what you need for this step.

a) $\mathcal{L} = \frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi + \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\phi^T\phi$ where $\phi \to \phi' = e^{ig\lambda\cdot\theta}\phi$

where λ are the 496 generators of SO(32)

b) $\partial_\mu \to D_\mu = \partial_\mu + ig\lambda\cdot A_\mu$ where we introduced 496 gauge fields

$\lambda\cdot A_\mu \to \lambda\cdot A'_\mu = e^{ig\lambda\cdot\theta}\lambda\cdot A_\mu e^{-ig\lambda\cdot\theta} + \frac{i}{g}\partial_\mu\left(e^{ig\lambda\cdot\theta}\right)e^{-ig\lambda\cdot\theta}$

c) $-\frac{1}{16\pi}F^a_{\mu\nu}F^{\mu\nu a}$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + igf^{abc}A^b_\mu A^c_\nu$

d)