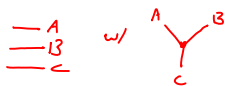



Mr. Feynman and Mr. Lagrangian

Recall the ABC's:

$$\mathcal{L} = \frac{i}{2} \partial_\mu \phi_A \partial^\mu \phi_A - \frac{1}{2} \left(\frac{\hbar c}{\lambda}\right)^2 \phi_A^4 + \frac{i}{2} \partial_\mu \phi_B \partial^\mu \phi_B - \frac{1}{2} \left(\frac{\hbar c}{\lambda}\right)^2 \phi_B^4 + \frac{i}{2} \partial_\mu \phi_C \partial^\mu \phi_C - \frac{1}{2} \left(\frac{\hbar c}{\lambda}\right)^2 \phi_C^4 - g \phi_A \phi_B \phi_C$$

In evaluating diagrams built from  w/  we used: $-ig$ vertex factor
 $\frac{i}{q^2 - \hbar^2 c^2}$ virtual particle propagators

There is actually a systematic way to extract the Feynman rules for a given Lagrangian, but to derive it requires QFT. We will simply state the results.

ABC example

Vertex factors: Write $i\mathcal{L}_{int}$ in momentum space ($i\hbar \partial_\mu \rightarrow p_\mu$)
 $\partial_\mu \rightarrow -\frac{i}{\hbar} p_\mu$

$$-ig \phi_A \phi_B \phi_C$$

Erase the field variables

$$-ig$$

Propagators: Write the relevant free-particle e.o.m. in momentum space and lose overall factor of \hbar
 $\partial_\mu \partial^\mu \phi + \left(\frac{\hbar c}{\lambda}\right)^2 \phi = 0 \Rightarrow [p^2 - (\hbar c)^2] \phi = 0$

Multiply the inverse of the term in brackets $\times i$

$$\frac{i}{p^2 - \hbar^2 c^2}$$

Who's house?! Dirac's house!

Consider the QED Lagrangian:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - mc\bar{\psi}\psi - q\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

The fundamental vertex is  or any other charged particle due to units carried by A_μ

and the vertex factor we get from $i\mathcal{L}_{int} = -iq\bar{\psi}\gamma^\mu\psi A_\mu$ removing $\bar{\psi}, \psi, \sqrt{\frac{4\pi}{q}}A_\mu$ giving: $-i\sqrt{\frac{4\pi}{q}}q\gamma^\mu = ig_c\gamma^\mu$

For the propagators we can have both virtual photons and electrons/positrons.

Electrons/Positrons we use: $i\gamma^\mu\partial_\mu\psi - \frac{mc}{\hbar}\psi = 0 \Rightarrow [\gamma^\mu p_\mu - mc] \psi = 0 \Rightarrow \frac{i(\gamma^\mu p_\mu + mc)}{p^2 - \hbar^2 c^2}$

For photons: Start w/ the massive Proca equation: $\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + (\frac{mc}{\hbar})^2 A^\nu = 0$

$$\partial_\mu \rightarrow -\frac{i}{\hbar} p_\mu \quad -\frac{i}{\hbar} p_\mu (-\frac{i}{\hbar} p^\mu A^\nu + \frac{i}{\hbar} p^\nu A^\mu) + (\frac{mc}{\hbar})^2 A^\nu = 0$$

We want this as one thing operating on A^ν so let's massage it a bit. First get rid of \hbar and i :

$$-p_\mu p^\mu A^\nu + p_\mu p^\nu A^\mu + (mc)^2 A^\nu = 0$$

Multiply by $\pi_{\lambda\nu}$ on both sides.

$$\pi_{\lambda\nu}(-p^2 A^\nu + p_\mu p^\nu A^\mu) + (mc)^2 \pi_{\lambda\nu} A^\nu = 0$$

$$\pi_{\lambda\nu}(-p^2 + (mc)^2) A^\nu + \underbrace{p_\mu p_\lambda A^\mu}_{\text{rename } \mu \rightarrow \nu \text{ since repeated}} = 0$$

$$[\pi_{\lambda\nu}(-p^2 + (mc)^2) + p_\nu p_\lambda] A^\nu = 0$$

Everywhere replace $\lambda \rightarrow \mu$

$$[\pi_{\mu\nu}(-p^2 + (mc)^2) + p_\mu p_\nu] A^\nu = 0$$

Useful for weak interactions later

For $m \neq 0$ the propagator is: $i[\mathcal{J}]^{-1} = \frac{-i}{p^2 - (mc)^2} \left[\pi_{\mu\nu} - \frac{p_\mu p_\nu}{(mc)^2} \right]$

For $m=0$ we also need transversality, i.e. $p_\nu A^\nu = 0$ leaving

$$[-\eta_{\mu\nu} p^2] A^\nu \Rightarrow i[\mathcal{J}]^{-1} = \frac{-i\eta_{\mu\nu}}{p^2} = \frac{-i\eta_{\mu\nu}}{p^2}$$

Photon propagator

QED (for now just charged leptons e, μ, τ and photons γ)

Diagrams are built from:  (or μ, τ versions)

In the ABC theory, all d.o.f. were scalars so: a) order was unimportant
b) we were guaranteed \mathcal{H} would be a scalar

In QED e, μ, τ are spinors and γ is a vector so: c) order is important
b) we have to be careful to ensure \mathcal{H} is a scalar

One new complication is that in most experiments we used unpolarized in-states and sum over all out states.
This means in calculating $|\mathcal{M}|^2$ we must a) average over incoming spin states
b) sum over outgoing spin states

Useful expressions:	Electron	Positron	Photon
Wavefunction	$\psi(x) = a e^{-\frac{i}{\hbar} p \cdot x} u^{(s)}(p)$	$\psi(x) = a e^{\frac{i}{\hbar} p \cdot x} v^{(s)}(p)$	$A_{\mu}(x) = a e^{-\frac{i}{\hbar} p \cdot x} \epsilon_{\mu}^{(s)}$ $s=1,2$
E.o.h. (non-space)	$(\gamma^{\mu} p_{\mu} - \hbar c)u = 0$ ^{DE}	$(\gamma^{\mu} p_{\mu} + \hbar c)v = 0$ ^{DE}	$\hat{p}^{\mu} \epsilon_{\mu} = 0, \epsilon^0 = 0$
Adjoint	$\bar{u} = u^{\dagger} \gamma^0$	$\bar{v} = v^{\dagger} \gamma^0$	$e^{\mu *}$
Adjoint E.o.h.	$\bar{u}(\gamma^{\mu} p_{\mu} - \hbar c) = 0$	$\bar{v}(\gamma^{\mu} p_{\mu} + \hbar c) = 0$	$p_{\mu} \epsilon^{\mu *} = 0$
Orthogonality	$\bar{u}^{(s)} u^{(s')} = 2\hbar c \delta_{ss'}$	$\bar{v}^{(s)} v^{(s')} = -2\hbar c \delta_{ss'}$	$\epsilon_{\mu}^{(s)} \epsilon^{\mu * (s')} = -\delta_{ss'}$
Completeness	$\sum_s u^{(s)} \bar{u}^{(s)} = \gamma^{\mu} p_{\mu} + \hbar c$	$\sum_s v^{(s)} \bar{v}^{(s)} = \gamma^{\mu} p_{\mu} - \hbar c$	$\sum_s \epsilon_{\mu}^{(s)} \epsilon_{\nu}^{(s)*} = \delta_{\mu\nu} - \hat{p}_{\mu} \hat{p}_{\nu}$

The Dirac spinors for $\vec{p} \neq 0$ in the new convention are:

$$u^{(1)} = \sqrt{\frac{E + \hbar c^2}{c}} \begin{pmatrix} 1 \\ 0 \\ \frac{c p_z}{E + \hbar c^2} \\ \frac{c(p_x + i p_y)}{E + \hbar c^2} \end{pmatrix} \quad u^{(2)} = \sqrt{\frac{E + \hbar c^2}{c}} \begin{pmatrix} 0 \\ 1 \\ \frac{c(p_x - i p_y)}{E + \hbar c^2} \\ -\frac{c p_z}{E + \hbar c^2} \end{pmatrix}$$

$$u^{(3)} = \sqrt{\frac{E + \hbar c^2}{c}} \begin{pmatrix} \frac{c(p_x - i p_y)}{E + \hbar c^2} \\ \frac{-c p_z}{E + \hbar c^2} \\ 0 \\ 1 \end{pmatrix} \quad u^{(4)} = -\sqrt{\frac{E + \hbar c^2}{c}} \begin{pmatrix} \frac{c p_z}{E + \hbar c^2} \\ \frac{c(p_x + i p_y)}{E + \hbar c^2} \\ 1 \\ 0 \end{pmatrix}$$

And for the spinor matrices:

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\text{w/ } \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$