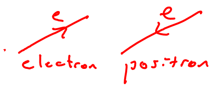
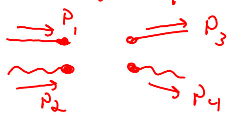
Monday, March 24, 2014
1:32 PM

Feynman Rules for QED

1. Draw diagram and label matter lines w/ arrows to distinguish particles/anti, e.g.

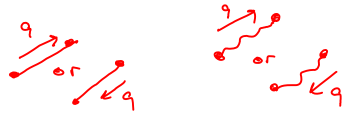


label momenta: a) external

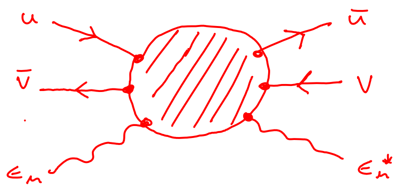


(along T)

b) internal



2. Each <u>external</u> line gets a factor according to:



Number factors w/ momenta, e.g. $\xrightarrow{p_1}$ $\Rightarrow$ $u(1)$

Note: Each factor is a 4spinor or 4vector.

3. Each vertex gets a factor $\quad i g_e \gamma^\mu \quad \left(g_e = e\sqrt{\frac{4\pi}{\hbar c}}\right)$ Note: Overall spin matrix.

4. Each <u>internal</u> line gets a factor: Electrons/Positrons $\quad \frac{i(\gamma^\mu q_\mu + mc)}{q^2 - m^2c^2}$ Overall spin matrix.

Photon $\quad -\frac{i\eta_{\mu\nu}}{q^2}$ Overall tensor
(μ,ν match vertex indices)

5. Conserve 4momentum at each vertex w/ $\quad (2\pi)^4 \delta^4(p_{in} + q_{in} - p_{out} - q_{out})$

6. Integrate over internal momenta w/ $\int \frac{d^4q}{(2\pi)^4}$ for each q.

7. Cancel overall $(2\pi)^4 \delta^4(p_{in} - p_{out})$ and $\times i$ to get $\mathcal{M}$.

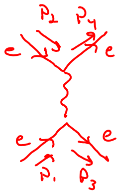
8. Antisymmetrize between diagrams related by switching 2 incoming electrons/positrons, 2 outgoing electrons/positrons or one incoming electron/positron with one outgoing positron/electron.

9. To get the order right for spinor elements (since we suppress indices) make "spinor sandwiches from matter lines" by starting w/ an outgoing matter particle line ($\bar{u}$) or incoming antimatter line ($\bar{v}$) and following along <u>only</u> matter segments writing vertex factors and internal matter propagators as we encounter them, until we eventually emerge on an outgoing antimatter (v) or incoming matter (u) line. The photon factors are less tricky since index notation gets them right.

Monday, March 24, 2014
1:58 PM

Example:

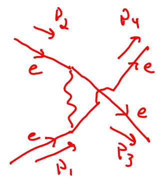
$e + e \to e + e$



$$\int \bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i\eta_{\mu\nu}}{q^2}\right) (2\pi)^4 \delta^4(p_1 - q - p_3)\, (2\pi)^4 \delta^4(p_2 + q - p_4) \frac{d^4 q}{(2\pi)^4}$$

$$\bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i\eta_{\mu\nu}}{(p_4 - p_2)^2}\right) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

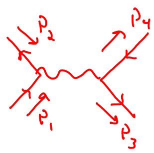
$$\Rightarrow \mathcal{M}_1 = \bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i\eta_{\mu\nu}}{(p_4 - p_2)^2}\right) i$$



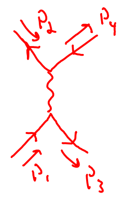
$\mathcal{M}_2 = \mathcal{M}_1\ (3 \leftrightarrow 4)$

By step 8: $\mathcal{M} = \mathcal{M}_1 - \mathcal{M}_2$

Consider $e^+ + e^- \to e^+ + e^-$:

1. 

$\mathcal{M}_1$

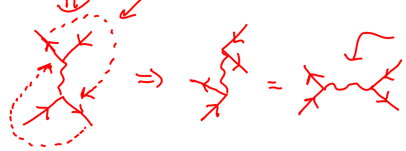
2. 

$\mathcal{M}_2$

$\mathcal{M} \overset{?}{=} \mathcal{M}_1 \pm \mathcal{M}_2$

$\Downarrow$

switch one outgoing e^- with one incoming e^+.



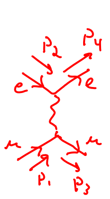
this is the same as in case 1.

So $\mathcal{M} = \mathcal{M}_1 - \mathcal{M}_2$

Tuesday, March 25, 2014
3:46 PM

Note: No contribution from 

Consider $e + \mu \to e + \mu$



$$\Rightarrow \quad \mathcal{M} = \bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i\eta_{\mu\nu}}{(p_4-p_2)^2}\right) i$$

Once we have $\mathcal{M}$, we can evaluate it given the following:

$u(i) = \alpha_i u^{(1)}(p_i) + \beta_i u^{(2)}(p_i) \qquad i = 1,2,3,4$

However in most experiments we average over $\alpha_1, \beta_1, \alpha_2, \beta_2$ and sum over $\alpha_3, \beta_3, \alpha_4, \beta_4$.

To do the avg/sum we first note that $\mathcal{M}$ contains terms of the form:

$$\mathcal{M} = \cdots \times \underbrace{\bar{u}(a)\, \overbrace{\Gamma_1}^{\text{some combination of spin matrices}}\, u(b)}_{\text{spinor sandwhich}}$$

Γ_1 — label to distinguish from other cases

Then:

$$|\mathcal{M}|^2 = \cdots \bar{u}(a)\, \Gamma_1\, u(b) \underbrace{[\bar{u}(a)\, \Gamma_2\, u(b)]}^{*} \cdots$$

Note: We give this a different label. In particular if Γ_1 carries spacetime indices μ,ν then Γ_2 will need different indices α,λ to avoid confusing contractions.

This thing is a number in spin space so we can freely transpose it to form †.

$$= \cdots \bar{u}(a)\, \Gamma_1\, u(b) \underbrace{[\bar{u}(a)\, \Gamma_2\, u(b)]^\dagger}$$

$$u(b)^\dagger\, \Gamma_2^\dagger\, \bar{u}(a)^\dagger = u(b)^\dagger\, \Gamma_2^\dagger \left(u(a)^\dagger \gamma^0\right)^\dagger$$

$$= u(b)^\dagger\, \Gamma_2^\dagger\, \gamma^{0\dagger} u(a)$$

insert $\gamma^0\gamma^0 = I$ and use $\gamma^0 = \gamma^{0\dagger}$

$$= u(b)^\dagger \gamma^0\gamma^0\, \Gamma_2^\dagger\, \gamma^0 u(a)$$

$$= \bar{u}(b)\, \bar{\Gamma}_2\, u(a)$$

$\bar{\Gamma}_2 = \gamma^0\, \Gamma_2^\dagger \gamma^0$

$$= \cdots \bar{u}(a)\, \Gamma_1\, u(b)\, \bar{u}(b)\, \bar{\Gamma}_2\, u(a)$$

Now to avg/sum the first step will be to sum so we can use $\sum_s u^{(s)} \bar{u}^{(s)} = \gamma^\mu p_\mu + mc = \not{p} + mc$

$$\sum_{s_b} |\mathcal{M}|^2 = \cdots \underbrace{\bar{u}(a)}_{\text{Row}}\, \underbrace{\Gamma_1\, (\not{p}_b + m_b c)\, \bar{\Gamma}_2}_{\text{Matrix}}\, \underbrace{u(a)}_{\text{Column}} \cdots$$

Now we can use that $RMC = \# = \mathrm{Tr}(MCR)$

$$\sum_{s_b} |\mathcal{M}|^2 = \cdots \mathrm{Tr}\left[\Gamma_1 (\not{p}_b + m_b c)\, \bar{\Gamma}_2\, u(a)\bar{u}(a)\right] \cdots$$

Now sum over s_a to get:

$$\sum_{s_a, s_b} |\mathcal{M}|^2 = \cdots \mathrm{Tr}\left[\Gamma_1 (\not{p}_b + m_b c)\, \bar{\Gamma}_2\, (\not{p}_a + m_a c)\right] \cdots$$