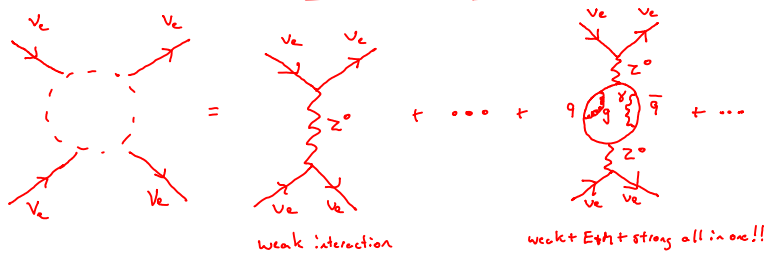


Hierarchy of interactivity: neutrinos - Weak
 charged leptons - $E+M$, Weak
 quarks - QCD, $E+M$, Weak

} "direct" interactions but...

In a given process \mathcal{M} must include all contributions!



Virtual states can bring in additional interactions!

Our next step will be to discuss QCD which means quarks, but we must also understand how QED plays out for quarks. We'll save the Weak interactions for last!

Quarks + QED

Quarks can replace e^\pm, μ^\pm, τ^\pm in any QED diagram w/ appropriate change of $g_e = e\sqrt{\frac{4\pi}{\hbar c}}$ (vertex factors).

Everything else same: $u, \bar{u}, \bar{d}, d, s, \bar{s}$ external states
spin-1/2 quarks γ_5

$$g = \frac{1}{3}e \text{ or } \frac{2}{3}e$$

propagators same, etc. (Note: color just along for ride)

Quarks + QCD

$$\mathcal{L} = \sum_{i=1}^6 \bar{\psi}_i \gamma^\mu D_\mu \psi_i + \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + \frac{1}{16\pi} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) - \frac{g}{16\pi} F^{ab} F^{cd} A_\mu^d A_\nu^e (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) - \frac{g}{16\pi} F^{abc} A^{\mu b} A^{\nu c} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) + \frac{g^2}{16\pi} F^{abc} F^{ade} A_\mu^b A_\nu^c A^{\mu d} A^{\nu e}$$

usual kinetic term

These are gluon-gluon interactions!

$$\psi_i = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}_i$$

$D_\mu \psi_i = \partial_\mu \psi_i + i g \lambda \cdot A_\mu \psi_i$
8 generators of SU(3)

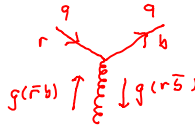
Color (r, g, b) plays role of "charges" and interactions via 8 gluons.

3 charges instead of 1 in QED!

Note in QED only one γ !

One practical complication is that QCD is defined in terms of quarks, but we only observe (and experiment w/) hadrons.

A useful way to think about the strong interactions is w/ vertices. In particular, it helps to think of the gluons as "bicolor" which could be expected since λ secretly carries 2 color indices λ_{ij} . However the gluons will always carry color-anticolor. Actually, it's a bit more complicated than this!



In fact since we often work with mesons ($q\bar{q}$) and gluons ($c\bar{c}$), it is useful to work with a linearly independent set of $c\bar{c}$ basis states:






"Octet"	$ 1\rangle = \frac{1}{\sqrt{2}}(r\bar{b} + b\bar{r})$	$ 5\rangle = -\frac{i}{\sqrt{2}}(r\bar{g} - g\bar{r})$	"Singlet"	$ 1\rangle = \frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$
	$ 2\rangle = -\frac{i}{\sqrt{2}}(r\bar{b} - b\bar{r})$	$ 6\rangle = \frac{1}{\sqrt{2}}(b\bar{g} + g\bar{b})$		Invariant under SU(3)
	$ 3\rangle = \frac{1}{\sqrt{2}}(r\bar{g} - g\bar{r})$	$ 7\rangle = -\frac{i}{\sqrt{2}}(b\bar{g} - g\bar{b})$		
	$ 4\rangle = \frac{1}{\sqrt{2}}(r\bar{g} + g\bar{r})$	$ 8\rangle = \frac{1}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g})$		

Rotated into each other under SU(3)
Note: |3>, |8> are "colorless" but nontrivial!

Quarks must exist in singlet combinations (for mesons).

Gluons only appear in Octet combinations. If a |1> gluon existed it could exist on its own and would behave much like γ . But we have never seen such a particle! Also w/ |1> $SU(3) \rightarrow U(1)$,
8-generators 9-generators

Let's compare:

	External State Labels	Internal Propagators	Vertex Factors
ABC	none	$\frac{i}{q^2 - m^2 c^2}$	 $-ig$
QED	u, \bar{u}, v, \bar{v} "matter"	$\frac{i(\not{q} + mc)}{q^2 - m^2 c^2}$	 $ig\gamma^\mu$
	$\epsilon_n, \epsilon^{*n}$ "photons"	$-\frac{i\eta_{\mu\nu}}{q^2}$	 $-\frac{ig_s}{2} \lambda^a \gamma^\mu$
QCD	$u_c, \bar{u}_c, v_c, \bar{v}_c$ "matter"	$\frac{i(\not{q} + mc)}{q^2 - m^2 c^2}$	 $-ig_s f^{abc} [\sigma_{\mu\nu}(k_1 - k_2)_\lambda + \sigma_{\nu\lambda}(k_2 - k_3)_\mu + \sigma_{\lambda\mu}(k_3 - k_1)_\nu]$
	$\epsilon_m^a, \epsilon_n^{*a}$ "gluons"	$-\frac{i\pi n^a m^b}{q^2}$	 $-ig_s^2 [f^{abc} f^{def} (\sigma_{\mu\nu}\sigma_{\rho\lambda} - \sigma_{\mu\rho}\sigma_{\nu\lambda}) + f^{abm} f^{cmn} (\sigma_{\mu\nu}\sigma_{\rho\lambda} - \sigma_{\mu\rho}\sigma_{\nu\lambda}) + f^{acm} f^{bcn} (\sigma_{\mu\nu}\sigma_{\rho\lambda} - \sigma_{\mu\rho}\sigma_{\nu\lambda})]$

Okay, so what are $c, a^\alpha, \lambda^a, f^{abc}$?

c : There are 3 colors so "charge space" is 3D w/


$c_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \text{red}$ $c_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \text{blue}$ $c_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \text{green}$ basis in H^3 space of color states.

An arbitrary state can have $c = a_i c_i$:

↑ complex coefficients, hence $c^\dagger = c^T^*$

a^α : There are 8 gluons w/

$a^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $a^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$... $a^8 = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ 1 \end{pmatrix}$ basis in H^8 space of gluon states.

Recall  naively $3 \times 3 = 9$ gluons, but the theory only includes 8 transformed into each other by $SU(3)$ (which has 8 generators!).

The "singlet" gluon $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$ does not exist (would look a lot like γ , and we haven't seen it!), Of course we haven't seen the other gluons, but for those we can use the confinement argument.

If the singlet did exist, then we would say $QCD \leftrightarrow U(3)$ (instead of $SU(3)$).

λ^a : For QED the γ_{ab}^μ matrices link "spin-space" to "space-time", i.e. 4 4×4 matrices.

The λ_{ij}^a matrices of QCD link H^3 of "color-space" to H^8 of "gluon-space", i.e. 8 3×3 matrices (8.34).

Just like w/ γ^a , we will leave off color-space labels and write λ^a (and c instead of c_i).

f^{abc} : $[\lambda^a, \lambda^b] = 2i f^{abc} \lambda^c$ structure constants of $SU(3)$ Lie Algebra

One huge technical complication that arises is for diagrams involving internal loops. In this case we have to be very careful not to count gauge equivalent (physically indistinct) configurations more than once. To get the counting right w/out losing gauge invariance, we introduce Faddeev-Popov ghosts which are additional nonphysical fields whose sole purpose is to cancel the nonphysical gauge equivalent fluctuations of the physical fields.